PAPER II

PART I

Answer **all** questions in this part.

1. Solve the equation $2^{2x} - 3(2^{x+2}) + 32 = 0$. **(8 marks)**

2. (a) Simplify $\frac{x-5}{6} - \frac{6}{x-5}$.

   (b) Find the gradient of a line joining the points $A(-4,-6)$ and $B(6,8)$. **(8 marks)**

3. A man travels 5 km due east of a town A to town B, and then travels 7 km on a bearing of 162° from B to another town C.
   Calculate the:

   (a) Shortest distance from A to C, correct to the nearest km
   (b) Bearing of C from A, correct to the nearest degree

   (8 marks)

4. **Table 2.1** gives the weights (in kg) of 35 students in a particular school.

Table 2.1: Weights of students

Weight (kg)	40-44	45-49	50-54	55-59	60-64	65-69	70-74
Frequency	5	6	10	4	5	3	2

   (a) Construct a cumulative frequency table for the distribution.
   (b) Draw a cumulative frequency curve for the distribution.
   (c) Use your curve to estimate the median. **(8 marks)**

5. (a) If $y = \left[\frac{1}{3}x^3 + 2x + 5\right]^2$, find $\frac{dy}{dx}$.

   (b) Given that $\log 3 = 0.4771$ and $\log 2 = 0.3010$,
   evaluate $\log 162 + \log 36 - \log 54$. **(8 marks)**

PART II

Answer five questions only in this part.

6. (a) Express the sum of 112012_{three} and 21121_{three} in base six.

   (b) If $\frac{8^{\frac{x}{6}} \times 16^{\frac{x}{12}}}{2^{\frac{3}{4}} \times 4^{\frac{3}{4}}} = 10^0$, find the value of x.

(12 marks)

7. (a) If $T = 2\pi\sqrt{\frac{L}{G}}$, make G the subject of the formula.
   Hence, find the value of G to the nearest whole number when
   $T = 45$, $L = 8$ and $\pi = 3.142$.

   (b) In a particular guest house, the cost per person (C) for an accommodation is partly constant and partly varies inversely as the number of guests (N). If the cost per person, for 24 persons is ₦800.00 and the cost per person, for 18 persons is ₦1200.00, find the cost per person, for 42 persons to the nearest naira.

(12 marks)

8. (a) A cylindrical metal pipe 3.5 m long has internal and external radii 5.0 cm and 7.5 cm respectively. Calculate the volume of:
   (i) Water that can fill the pipe in litres
   (ii) Metal in the pipe in cm^3 2

   (b) A sector of a circle with radius 12 cm and angle 240° is used to form a cone. Calculate the:
   (i) Radius of the cone
   (ii) Vertical angle of the cone, correct to two decimal places

(12 marks)

9. P is a point on latitude 28° 26' N and longitude 105° 34' W. R is on the same latitude as P but lies on longitude 34° 26' E. Calculate, correct to three significant figures, the:
   (a) Distance between P and R along the latitude
   (b) Length of the chord PR
   (Take $\pi = 3.14$, $R = 6,400$ km)

(12 marks)

10. (a) Draw the graph of $y = x^2 - 3x + 2$ for $-2 \leq x \leq 4$, using a scale of 2 cm to 1 unit on x - axis and 2 cm to 2 units on y-axis.

(b) From your graph, determine the:

(i) Equation of the line of symmetry of the curve
(ii) Roots of the equation $x^2 - 3x + 2 = 0$
(iii) Gradient of the curve at $x = 0$
(iv) Minimum value of y **(12 marks)**

11. (a) Towns P(lat. 40°N, long.20°W) and Q(lat.40°N, long.70°W) are two places on the Earth's surface. Calculate, correct to 2 decimal places, the:

(i) Radius of the parallel of latitude on which P and Q lie
(ii) Shortest distance PQ along the parallel of latitude
(iii) Time taken for an aeroplane to cover the shortest distance PQ at an average speed of 800 km/h. (Use $\pi = \frac{22}{7}$ and R = 6,400 km)

(b) Simplify $\frac{2(x^2+x-6)}{3x^2-27} \div \frac{x^2-x-2}{x-3}$. **(12 marks)**

12. **Table 2.2** shows the scores of candidates in an entrance examination into a college.

Table 2.2: Scores of candidates

Marks(%)	21-30	31 - 40	41 - 50	51 - 60	61 - 70	71 - 80	81 - 90
No. of students	9	20	31	42	50	32	16

(a) Prepare a cumulative frequency table and draw the cumulative frequency curve for the distribution.

(b) Use your curve to estimate the:
(i) Median mark
(ii) Upper and lower quartiles
(iii) Semi-inter quartile range

(12 marks)

QUESTION 1

$2^{2x} - 3(2^{x+2}) + 32 = 0$

$(2^x)^2 - 3(2^x \times 2^2) + 32 = 0$

$(2^x)^2 - 12(2^x) + 32 = 0$

$(2^x)^2 - 8(2^x) - 4(2^x) + 32 = 0$

$2^x[2^x - 8] - 4[2^x - 8] = 0$

substitute whatever term

$(2^x - 4)(2^x - 8) = 0$

$2^x - 4 = 0$ or $2^x - 8 = 0$

$2^x = 4$ or $2^x = 8$

$2^x = 2^2$ or $2^x = 2^3$

$x = 2$ or $x = 3$

QUESTION 2

(a) $\frac{x-5}{6} - \frac{6}{x-5} = \frac{(x-5)^2 - 6^2}{6(x-5)}$

$= \frac{(x-5+6)(x-5-6)}{6(x-5)}$

$= \frac{(x+1)(x-11)}{6(x-5)}$

or $= \frac{x^2 - 10x - 11}{6(x-5)}$

(b) Gradient, $m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{8-(-6)}{6-(-4)}$

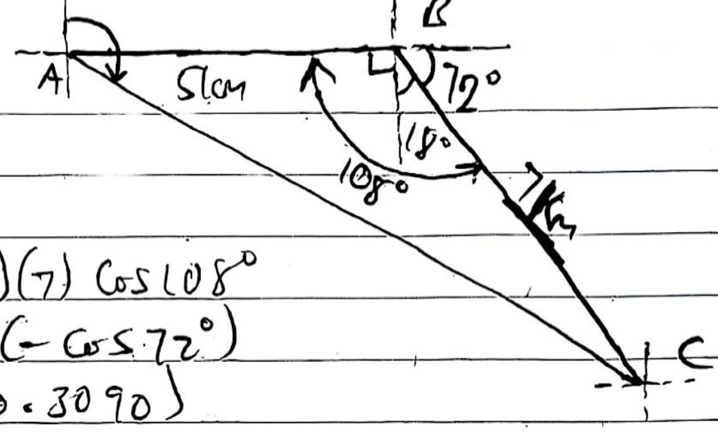
$= \frac{8+6}{6+4}$

$= \frac{14}{10}$

$= 7/5$ or 1.4

QUESTION 3

(a)



$\overline{AC}^2 = 5^2 + 7^2 - 2(5)(7)\cos 108°$

$\overline{AC}^2 = 25 + 49 - 70(-\cos 72°)$

$\overline{AC}^2 = 74 + 70(0.3090)$

$\overline{AC}^2 = 74 + 21.63$

$\overline{AC} = \sqrt{95.63}$

$\overline{AC} = 9.779 \approx 10km$

(b) Using sine rule.

$$\frac{7}{\sin A} = \frac{9.779}{\sin 108}$$

$$\sin A = \frac{7\sin 108}{9.779} = \frac{7\sin 72°}{9.779}$$

$$\sin A = \frac{7 \times 0.9511}{9.779}$$

$\sin A = 0.6808$

$A = \sin^{-1}(0.6808)$

$A = 42.9$

$A \approx 43°$

Bearing of C from A $= 90° + 43°$

$= 133°$

QUESTION 4

(a)

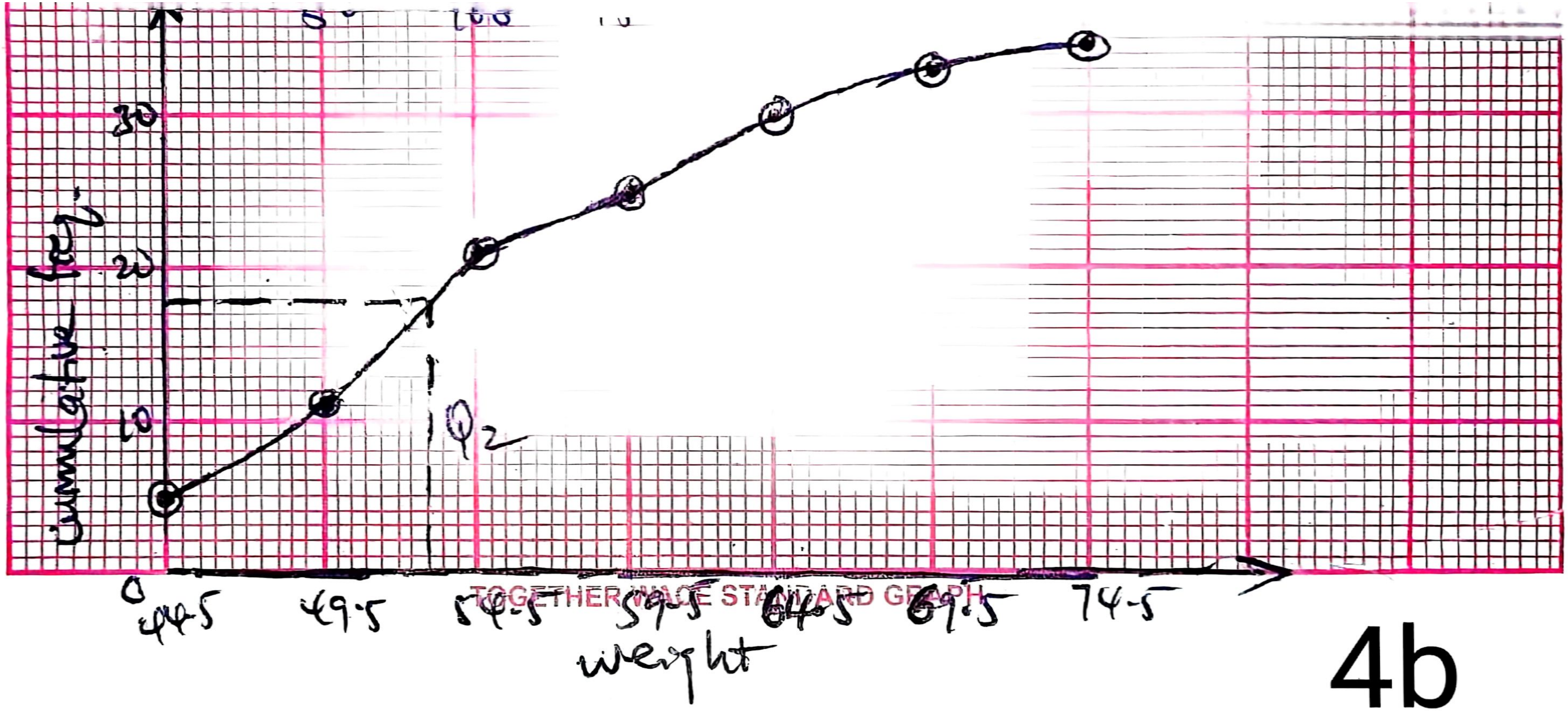
Weight (kg)	freq, f	upper class boundary	Cumulative freq. c.f
40–44	5	44.5	5
45–49	6	49.5	11
50–54	10	54.5	21
55–59	4	59.5	25
60–64	5	64.5	30
65–69	3	69.5	33
70–74	2	74.5	35

(c) Median = $\frac{1}{2}(35+1)$ th position

$\frac{1}{2}(36)$ th position

18th mark

median, Q_2 = 53 kg.



cumulative freq
30
20
10
0
44.5
49.5
54.5
59.5
64.5
69.5
74.5
weight
Q_2
TOGETHER MADE STANDARD GRAPH

4b

QUESTION 5

ⓐ $y = \left[\frac{1}{3}x^3 + 2x + 5\right]^2$

let $u = \frac{1}{3}x^3 + 2x + 5$

$du/dx = \frac{1}{3}(3)x^2 + 2 = x^2 + 2$

$y = u^2$

$dy/du = 2u$

$dy/dx = \frac{dy}{du} \times \frac{du}{dx} = 2u \times (x^2+2)$

$\frac{dy}{dx} = 2\left[\frac{1}{3}x^3 + 2x + 5\right]\left[x^2+2\right]$

$= 2(x^2+2)\left(\frac{1}{3}x^3 + 2x + 5\right)$

ⓑ $\log 162 + \log 36 - \log 54$

$= \log 162 - \log 54 + \log 36$

$= \log\left(\frac{162}{54}\right) + \log 36$

$= \log 3 + \log 36$

$= \log 3 + \log 6^2$

$= \log 3 + 2\log 6$

$= \log 3 + 2\left[\log 3 + \log 2\right]$

$= \log 3 + 2\log 3 + 2\log 2$

$= 3\log 3 + 2\log 2$

$= 3(0.4771) + 2(0.3010)$

$= 1.4313 + 0.602$

2.0333

QUESTION 6

(a)

$$\begin{array}{r} {}^{1}1\ {}^{1}1\ 2\ {}^{1}0\ {}^{1}1\ 2_{three} \\ +\quad 2\ 1\ 1\ 2\ 1_{three} \\ \hline 2\ 1\ 0\ 2\ 1\ 0_{three} \end{array}$$

$210210_{three} = (2\times3^5) + (1\times3^4) + (0\times3^3) + (2\times3^2) + (1\times3^1) + (0\times3^0)$

$= 486 + 81 + 0 + 18 + 3 + 0$

$= 588_{ten}$

$588_{ten} =$

$$\begin{array}{r|l} 6 & 588 \\ 6 & 98\ R\ 0 \\ 6 & 16\ R\ 2 \\ 6 & 2\ R\ 4 \\ & 0\ R\ 2 \end{array}$$

$\therefore$ Sum $= 2420_{six}$

(b) $\dfrac{8^{x/6} \times 16^{x/12}}{2^{3/8} \times 4^{3/4}} = 1$

$\dfrac{[2^3]^{x/6} \times [2^4]^{x/12}}{2^{3/8} \times [2^2]^{3/4}} = 1$

$\dfrac{2^{x/2} \times 2^{x/3}}{2^{3/8} \times 2^{3/2}} = 1$

$\dfrac{2^{x/2 + x/3}}{2^{3/8 + 3/2}} = 1$

$\dfrac{2^{5x/6}}{2^{15/8}} = 1$

i.e

$2^{5x/6} = 2^{15/8}$

$\dfrac{5x}{6} = \dfrac{15}{8}$

$x = 6/5 \times 15/8$

$x = \dfrac{18}{8}$

$x = 9/4$

$x = 2\frac{1}{4}$

QUESTION 7

(a)

$$T = 2\pi\sqrt{\frac{L}{G}}$$

square both sides

$$T^2 = 4\pi^2 \frac{L}{G}$$

$$G = \frac{4\pi^2 L}{T^2}$$

when: $T = 45$, $L = 8$ and $\pi = 3.142$

$$G = \frac{4 \times (3.142)^2 \times 8}{45^2}$$

$$G = 0.156$$

$G \simeq 0$ (nearest whole number)

(b) Given: $C = a + \frac{b}{N}$ where a & b are constants

Given: $C = ₦800$ and $N = 24$

$$800 = a + \frac{b}{24} \quad \text{——— (1)}$$

Given: $C = ₦1200$ and $N = 18$

$$1200 = a + \frac{b}{18} \quad \text{——— (2)}$$

subtract equ (1) from equ (2)

$$400 = b\left(\frac{1}{18} - \frac{1}{24}\right)$$

$$400 = b\left(\frac{4-3}{72}\right)$$

$$400 = \frac{b}{72}$$

$$b = 28800$$

put this in equ (1)

$$800 = a + \frac{28800}{24}$$

$$a = 800 - 1200$$

$$a = -400$$

Equation: $C = -400 + \frac{28800}{N}$

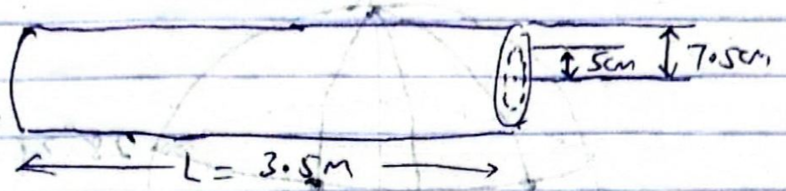
When: $N = 42$

$$C = -400 + \frac{28800}{42} = -400 + 685.71$$

$= ₦286$ (nearest naira)

QUESTION 8

(a)



(ii) water that can fill the pipe = volume of inner cylinder

$= \pi r^2 l$

$= \frac{22}{7} \times \frac{5}{1} \times \frac{5}{1} \times 350$

$= 27500 \text{ cm}^3$

$= 27.5$ litres

Tinashe solution team

(ii) Volume of metal in the pipe $= \pi (R^2 - r^2) l$

$= \frac{22}{7} \times (7.5^2 - 5.0^2) \times 350$

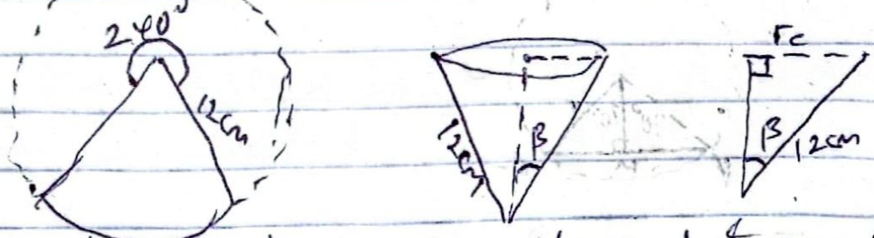
$= \frac{22}{7} \times (7.5 + 5.0)(7.5 - 5.0) \times 350$

$= \frac{22}{7} \times 12.5 \times 2.5 \times 350$

$= 34375 \text{ cm}^3$

(b)

(i)



Arc length of sector = circular part of cone

$\frac{\theta}{360°} \times 2\pi r_s = 2\pi r_c$

$\frac{240}{360} \times 2 \times 12 = 2 r_c$

$16 = 2 r_c$

radius of cone, $r_c = 16/2 = 8$cm

NB: radius of sector = slant height of cone

(ii) Vertical angle $2\beta = 2 \times \sin^{-1}(r_c / l)$

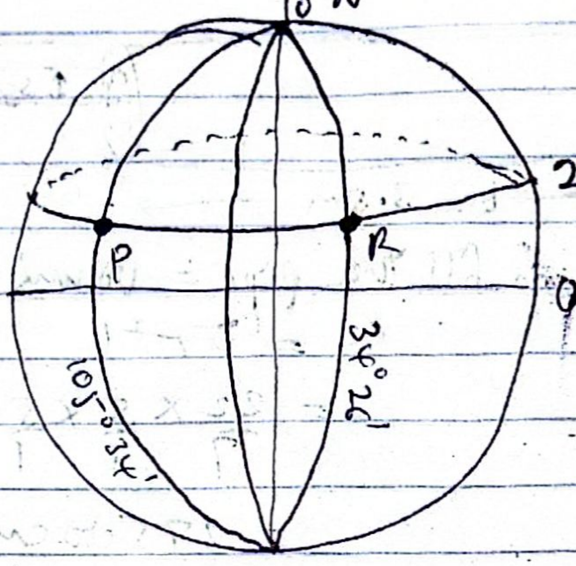
$= 2 \times \sin^{-1}(8/12)$

$= 2 \times \sin^{-1}(2/3)$

$= 2 \times 41.81$

$= 83.62°$

QUESTION 9



(a) $\overline{PR} = \frac{\theta}{360^\circ} \times 2\pi R\cos\phi$

where: $\theta = 105^\circ 34' + 34^\circ 26' = 140^\circ 00'$

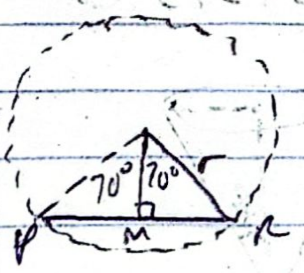
$\phi = 28^\circ 26' = 28.43^\circ$

$\overline{PR} = \frac{140^\circ}{360^\circ} \times 2 \times 3.14 \times 6400 \times \cos 28.43$

$= 13744.775$

$= 13700\text{km} (3\text{ s.f})$

(b)



$\overline{MR} = r\sin 70^\circ = R\cos\phi \sin 70^\circ$

Length of chord, $\overline{PR} = 2R\cos\phi \sin 70^\circ$

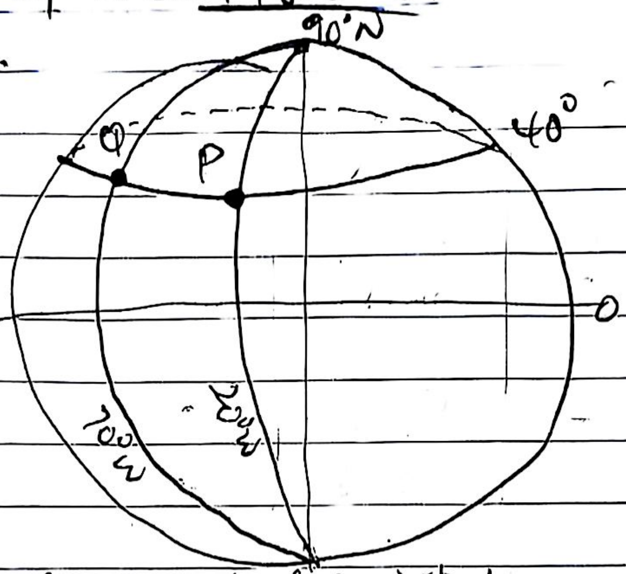
$= 2 \times 6400 \times \cos 28.43 \times \sin 70^\circ$

$= 10577.14$

$= 10600\text{ km} (3.s.f)$

QUESTION 11

(a)



(i) Radius of parallel of latitude $r = R\cos\phi$

$= 6400 \times \cos 40^\circ$

$= 4902.68$ km

(ii) $PQ = \frac{\theta}{360} \times 2\pi r$

$= \frac{70-20}{360} \times \frac{2}{1} \times \frac{22}{7} \times 4902.68$

$= 4280.12$ km

(iii) time, $t = \frac{\text{distance } PQ}{\text{speed}} = \frac{4280.12}{800} = 5.35$ hours

(b) $\frac{2(x^2+x-6)}{3x^2-27} \div \frac{x^2-x-2}{x-3}$

$= \frac{2(x+3)(x-2)}{3(x^2-9)} \div \frac{(x-2)(x+1)}{x-3}$

$= \frac{2(x+3)(x-2)}{3(x+3)(x-3)} \times \frac{(x-3)}{(x-2)(x+1)}$

$= \frac{2}{3(x+1)}$

QUESTION 1.2

(a)

Marks (%)	freq. f	Upper class boundary	Cumulative freq. c.f
21 – 30	9	30.5	9
31 – 40	20	40.5	29
41 – 50	31	50.5	60
51 – 60	42	60.5	102
61 – 70	50	70.5	152
71 – 80	32	80.5	184
81 – 90	16	90.5	200

(b) (i) Median $= \frac{1}{2}(n+1)$th mark

$= \frac{1}{2}(200+1)$th mark

$= 100.5$th mark

Median, $Q_2 = 60.5$ marks

Tabulate solution here

(ii) lower quartile, $Q_1 = \frac{1}{4}(200+1)$th mark

$= 50.25$ mark

$Q_1 = 48$ marks

Upper quartile, $Q_3 = \frac{3}{4}(200+1)$ mark

$= 150.75$ mark

$Q_3 = 70.5$ marks

(iii) Semi-inter quartile range, $Q = \frac{Q_3 - Q_1}{2}$

$Q = \frac{70.5 - 48}{2}$

$Q = 11.25$

QUESTION 12

GRAPH