

PAPER II

PART I

Attempt all questions in this part.

1. (a) Calculate the simple interest on ₦720,000.00 for 3 years at $5\frac{1}{2}\%$ per annum. Hence, find the amount at the end of 3 years.
- (b) In a class of 75 students, the ratio of boys to girls is 2:3. How many boys should be added to the class in order to get a ratio 7:9?

(8 marks)

2. (a) Solve the equation $\frac{3}{a} = \frac{2}{a+1} + \frac{3}{3a-1}$

- (b) Simplify $\frac{x-y}{y+x} + \frac{y-x}{x+y}$

(8 marks)

3. A man travels 5 km due east of a town A to town B, and then travels 7 km on a bearing of 162° from B to another town C. Calculate the:

- (a) Shortest distance from A to C, correct to the nearest km
- (b) Bearing of C from A, correct to the nearest degree

(8 marks)

4. (a) Simplify $\frac{8}{3r^2-48} - \frac{4}{3r^2-12r}$.

- (b) Evaluate $\int (3x^2+2)^2 dx$.

(8 marks)

5. (a) A box contains 10 mangoes and 9 bananas. If 2 fruits are picked from the box without replacement, find the probability that both are of the same fruits.

- (b) Use elimination method to solve the simultaneous equations;
 $2x + y = 17$, $x + 2y = 2$.

(8 marks)

PART II

Answer five questions only in this part.

6. (a) Use logarithm table to evaluate $\frac{53.76 \times 0.536}{(0.124)^2 \times 0.325}$.

(b) Solve the equation $\frac{3}{(x+2)} - \frac{1}{2(x-3)} = \frac{5}{x-3}$.

(12 marks)

7. (a) John has more money than Samson. If John gives ₦500.00 to Samson, they would have the same amount. If Samson gives John ₦1,200.00, John will have twice as much as Samson. How much does each of them have?

(b) Given that $2a - 3b = 5$, solve the quadratic equation $b^2 - a = 2$.

(12 marks)

8. (a) A cylindrical metal pipe 3.5 m long has internal and external radii 5.0 cm and 7.5 cm respectively. Calculate the volume of:
- Water that can fill the pipe in litres
 - Metal in the pipe in cm^3

- (b) A sector of a circle with radius 12 cm and angle 240° is used to form a cone. Calculate the:
- Radius of the cone
 - Vertical angle of the cone, correct to two decimal places

(12 marks)

9. (a) In an Arithmetic Progression (A.P.), the difference between the 14th and 7th term is 56 and the 12th term is $2\frac{1}{3}$ of the 4th term. Find the:
- Common difference
 - Sum of the first nine terms of the A.P.

(b) Evaluate $\int_{-4}^1 \left(\frac{1}{2} - 2x\right) dx$.

(12 marks)

10. Two cities X and Y are on latitude 38°S and their longitudes are 50°E and 130°W respectively. Calculate and correct to 3 significant figures the:

- Speed of X due to the rotation of the earth
 - Distance between X and Y along the great circle
 - Distance between X and Y along the parallel of latitude
- (Take $\pi = \frac{22}{7}$ and $R = 6400 \text{ km}$)

(12 marks)

11. (a) Copy and complete Table 2.1 for the relation $y = 2x^2 - 5x - 6$, for $-2 \leq x \leq 5$.

Table 2.1: Values of x and y

x	-2	-1	0	1	2	3	4	5
y	12					-3		19

- (b) Using a scale of 2 cm to represent 1 unit on x -axis and 2 cm to represent 5 units on y -axis, draw the graph of $y = 2x^2 - 5x - 6$ for $-2 \leq x \leq 5$.

- (c) Use your graph to find the:

- Roots of the equation $2x^2 - 5x - 6 = 0$, correct to one decimal place
- Values of x for which $2x^2 - 5 = 8$, correct to one decimal place
- Gradient of the tangent at the point where $x = 2$

(12 marks)

12. Table 2.2 shows the scores of candidates in an entrance examination into a college.

Table 2.2: Scores of candidates

Marks(%)	21-30	31-40	41-50	51-60	61-70	71-80	81-90
No. of students	9	20	31	42	50	32	16

- Prepare a cumulative frequency table and draw the cumulative frequency curve for the distribution.
- Use your curve to estimate the:
 - Median mark
 - Upper and lower quartiles
 - Semi-inter quartile range

(12 marks)

Question 1

$$\textcircled{a} \text{ Simple Interest} = \frac{PRT}{100} = \frac{720,000 \times 5.5 \times 3}{100}$$

$$I = 118,800$$

$$\begin{aligned} \text{Amount} &= \text{Principal} + \text{Interest} \\ &= 720,000 + 118,800 \\ &= 838,800 \end{aligned}$$

$$\textcircled{b} \text{ No of boys} = \frac{2}{5} \times 75 = 2 \times 15 = 30 \text{ boys}$$

Hence: there are $75 - 30 = 45$ girls

$$\frac{P+30}{45} = \frac{7}{9}$$

$$P+30 = \frac{7}{9} \times 45 = 35$$

$$P = 35 - 30 = 5$$

5 boys must be added.

Question 2

(a) $\frac{3}{a} = \frac{2}{a+1} + \frac{3}{3a-1}$

multiply through by l.c.m: $a(a+1)(3a-1)$

$$3(a+1)(3a-1) = 2a(3a-1) + 3a(a+1)$$

$$3[3a^2 + 2a - 1] = 6a^2 - 2a + 3a^2 + 3a$$

$$9a^2 + 6a - 3 = 9a^2 + a$$

$$6a - 3 = a$$

$$5a = 3$$

$$a = \frac{3}{5}$$

(b) $\frac{x-y}{y+x} \div \frac{y-x}{x+y} = \frac{x-y}{y+x} \times \frac{x+y}{y-x}$

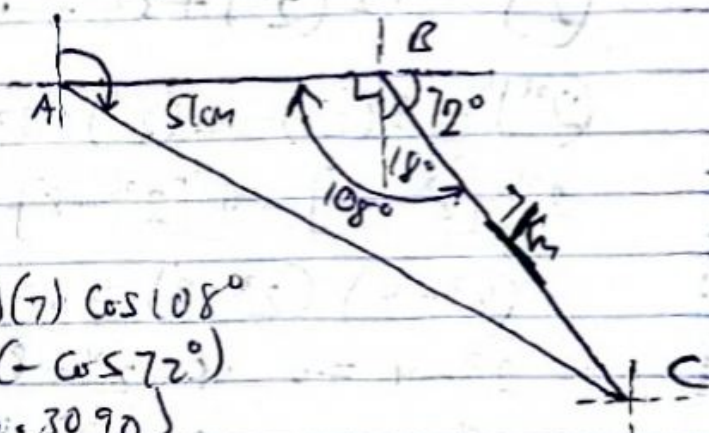
$$= \frac{x-y}{y-x}$$

$$= -\frac{(y-x)}{y-x}$$

$$= -1$$

QUESTION 3

(a)



$$AC^2 = 5^2 + 7^2 - 2(5)(7) \cos 108^\circ$$

$$AC^2 = 25 + 49 - 70 (-\cos 72^\circ)$$

$$AC^2 = 74 + 70(0.3090)$$

$$AC^2 = 74 + 21.63$$

$$AC = \sqrt{95.63}$$

$$AC = 9.779 \approx 10 \text{ km}$$

(b) Using sine rule.

$$\frac{1}{\sin A} = \frac{9.779}{\sin 108}$$

$$\sin A = \frac{7 \sin 108}{9.779} = \frac{7 \sin 72^\circ}{9.779}$$

$$\sin A = \frac{7 \times 0.9511}{9.779}$$

$$\sin A = 0.6808$$

$$A = \sin^{-1}(0.6808)$$

$$A \approx 42.9^\circ$$

$$A \approx 43^\circ$$

$$\text{Bearing of C from A} = 90^\circ + 43^\circ = 133^\circ$$

sin rule

Question 4

$$\begin{aligned} \textcircled{9} \quad & \frac{8}{3r^2-48} - \frac{4}{3r^2-12r} \\ &= \frac{8}{3(r^2-16)} - \frac{4}{3r(r-4)} \\ &= \frac{8}{3(r+4)(r-4)} - \frac{4}{3r(r-4)} \\ &= \frac{8r - 4(r+4)}{3r(r+4)(r-4)} \\ &= \frac{8r - 4r - 16}{3r(r+4)(r-4)} \\ &= \frac{4r - 16}{3r(r+4)(r-4)} \\ &= \frac{4(r-4)}{3r(r+4)(r-4)} \\ &= \frac{4}{3r(r+4)} \end{aligned}$$

$$\begin{aligned} \textcircled{10} \quad & \int (3x^2+2)^2 dx = \int (3x^2+2)(3x^2+2) dx \\ &= \int (9x^4 + 12x^2 + 4) dx \\ &= \frac{9x^5}{5} + \frac{12x^3}{3} + 4x + C \\ &= \frac{9x^5}{5} + 4x^3 + 4x + C \end{aligned}$$

Question 5

④ Total no of fruits = $10 + 9 = 19$

prob (mango) = $\frac{10}{19}$ & prob (banana) = $\frac{9}{19}$

prob (both same fruit) = prob (both mango) + prob (both banana)

$$= \left(\frac{10}{19} \times \frac{9}{18} \right) + \left(\frac{9}{19} \times \frac{8}{18} \right)$$

$$= \frac{90}{342} + \frac{72}{342}$$

$$= \frac{162}{342} = \frac{81}{171}$$

⑥ $2x + y = 17$ ————— ①

$3x + 2y = 2$ ————— ②

multiply eqn ① by "2"

$4x + 2y = 34$ ————— ③

subtract eqn ② from eqn ③

$x = 32$

put $x = 32$ into eqn ①

$2(32) + y = 17$

$64 + y = 17$

$y = 17 - 64 = -47$

QUESTION 7

- (a) let John's money be "a" naira
& Samson's be "b" naira

$$a - 500 = b + 500$$

$$a - b = 1000 \quad \text{--- (1)}$$

Also $a + 1200 = 2(b - 1200)$

$$a + 1200 = 2b - 2400$$

$$a - 2b = -3600 \quad \text{--- (2)}$$

subtract eqn (2) from eqn (1)

$$-b - (-2b) = 1000 - (-3600)$$

$$b = 4600$$

put this into eqn (1)

$$a - 4600 = 1000$$

$$a = 4600 + 1000$$

$$a = 5600$$

John had ~~5600~~ 5600.

Samson had ~~4600~~ 4600.

(b) Given: $2a - 3b = 5$

i.e. $2a = 3b + 5$

$$a = \frac{3b + 5}{2}$$

$$b^2 - a = 2 \Rightarrow b^2 - \left[\frac{3b + 5}{2} \right] = 2$$

$$2b^2 - (3b + 5) = 4$$

$$2b^2 - 3b - 5 = 4$$

$$2b^2 - 3b - 9 = 0$$

$$2b^2 - 6b + 3b - 9 = 0$$

$$2b(b - 3) + 3(b - 3) = 0$$

$$(2b + 3)(b - 3) = 0$$

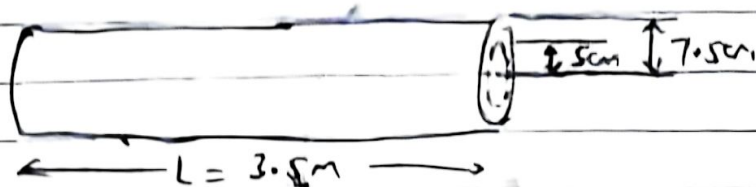
$$b = -\frac{3}{2} \text{ or } b = 3$$

If $b = -\frac{3}{2}$; $a = \frac{3(-\frac{3}{2}) + 5}{2} = \frac{1}{4}$

If $b = 3$; $a = \frac{3(3) + 5}{2} = 7$

QUESTION 8

(a)



(ii) water that can fill the pipe = volume of inner cylinder

$$= \pi r^2 l$$

$$= \frac{22}{7} \times \frac{5}{1} \times \frac{5}{1} \times 350$$

$$= 27500 \text{ cm}^3$$

$$= 27.5 \text{ litres}$$

(iii) Volume of metal in the pipe = $\pi (R^2 - r^2) l$

$$= \frac{22}{7} \times (7.5^2 - 5.0^2) \times 350$$

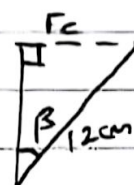
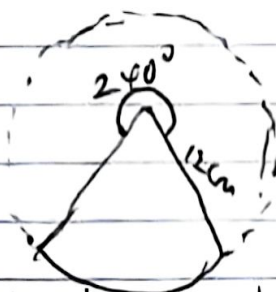
$$= \frac{22}{7} \times (7.5 + 5.0) (7.5 - 5.0) \times 350$$

$$= \frac{22}{7} \times 12.5 \times 2.5 \times 350$$

$$= 34375 \text{ cm}^3$$

(b)

(i)



Arc length of sector = circular part of cone

$$\frac{\theta}{360} \times 2\pi r_s = 2\pi r_c$$

$$\frac{240}{360} \times 2 \times 12 = 2r_c$$

$$16 = 2r_c$$

$$\text{radius of cone, } r_c = 16/2 = 8\text{cm}$$

NB:
radius of sector
= slant height
of cone

(ii) Vertical angle $2\beta = 2 \times \sin^{-1} \left(\frac{r_c}{l} \right)$

$$= 2 \times \sin^{-1} \left(\frac{8}{12} \right)$$

$$= 2 \times \sin^{-1} \left(\frac{2}{3} \right)$$

$$= 2 \times 41.81$$

$$= 83.62^\circ$$

QUESTION 9

(a) For A.P. $T_n = a + (n-1)d$

(i) $T_{13} - T_7 = 56$

$$a + 12d - a - 6d = 56$$

$$7d = 56$$

$$d = 56/7 = 8$$

Common difference = 8

(ii) $T_{12} = 2\frac{1}{3} \times T_4$

$$a + 11d = \frac{7}{3}(a + 3d)$$

$$a + 11(8) = \frac{7}{3}(a + 3(8))$$

$$a + 88 = \frac{7a + 56}{3}$$

$$\frac{7a}{3} - a = 88 - 56$$

$$\frac{4a}{3} = 32$$

$$a = \frac{3}{4} \times 32 = 24$$

1.5 marks solution tree

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$S_9 = \frac{9}{2} [2(24) + 8(8)]$$

$$= \frac{9}{2} [48 + 64]$$

$$= 504$$

(b) $\int_{-4}^1 (\frac{1}{2} - 2x) dx$

$$= \left[\frac{1}{2}x - x^2 \right]_{-4}^1$$

$$= \left[\frac{1}{2} - 1 \right] - \left[\frac{1}{2}(-4) - (-4)^2 \right]$$

$$= \left(-\frac{1}{2} \right) - (-2 - 16)$$

$$= -\frac{1}{2} + 18$$

$$= 17\frac{1}{2}$$

QUESTION 10

(a) linear speed, $v = \text{angular speed} \times \text{radius of latitude}$

$$v = \frac{2\pi (\text{radius})}{24 (\text{hours})} \times R \cos \phi$$

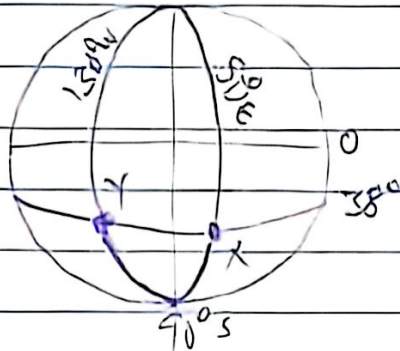
$$v = \frac{2\pi \times 6400 \times \cos 38^\circ}{24} \quad \text{where } \phi = 38^\circ$$

$$v = \frac{2 \times 22 \times 6400 \times 0.7880}{24 \times 7}$$

$$v = 1320.8 \text{ km/hr}$$

$$v = 1320 \text{ km/hr} \quad (3 \text{ s.f.})$$

(b)



$$\theta = 50 + 130 = 180^\circ$$

Central angle between X and Y

$$\cos C = \sin \phi_1 \sin \phi_2 + \cos \phi_1 \cos \phi_2 \cos \theta$$

$$= \sin(-38^\circ) \sin(-31^\circ) + \cos(-38^\circ) \cos(-31^\circ) \cos 180^\circ$$

$$= (-0.6157)(-0.6157) + (0.788)(0.788)(-1)$$

$$= -0.2418$$

$$C = \cos^{-1}(-0.2418) = 104^\circ$$

$$\text{Distance} = \frac{C}{360} \times 2\pi R = \frac{104}{360} \times 2 \times \frac{22}{7} \times 6400$$

$$= 11620 \text{ km}$$

$$= 11600 \text{ km} \quad (3 \text{ s.f.})$$

(c) Parallel of latitude

$$\overline{XY} = \frac{180}{360} \times \frac{2}{1} \times \frac{22}{7} \times 6400 \cos 31^\circ$$

$$= \frac{22 \times 6400 \times 0.788}{7} = 15850 \text{ km}$$

$$= 15900 \text{ km} \quad (3 \text{ s.f.})$$

QUESTION 12

⑥ Marks (%)	freq. f	upper class boundary	Cumulative freq. C.F
21-30	9	30.5	9
31-40	20	40.5	29
41-50	31	50.5	60
51-60	42	60.5	102
61-70	50	70.5	152
71-80	32	80.5	184
81-90	16	90.5	200

⑥ (i) Median = $\frac{1}{2}(n+1)$ th mark
 $= \frac{1}{2}(200+1)$ th mark
 $= 100.5$ th mark

Median, $Q_2 = 60.5$ marks

(ii) Lower quartile, $Q_1 = \frac{1}{4}(200+1)$ th mark
 $= 50.25$ mark

$Q_1 = 48$ marks

Upper quartile, $Q_3 = \frac{3}{4}(200+1)$ th mark
 $= 150.75$ mark

$Q_3 = 70.5$ marks

⑦ Semi-inter quartile range, $Q = \frac{Q_3 - Q_1}{2}$

$Q = \frac{70.5 - 48}{2}$

$Q = 11.25$