PAPER II

PART I

Answer all questions in this part.

1. If $x = \frac{2a+3}{3a-2}$, express $\frac{x-1}{2x+1}$ in terms of a. **(8 marks)**

2. A sphere has a volume of 300 cm^3. Another sphere has a radius which is one-third the radius of the first sphere. Find the volume of the second sphere. (Take $\pi = \frac{22}{7}$) **(8 marks)**

3. (a) Simplify $\frac{8}{3r^2-4t} - \frac{4}{3r^2-12r}$.

   (b) Evaluate $\int (3x^2+2)^2 \, dx$

   (8 marks)

4. A particle moves in a straight line such that the distance travelled is specified by the equation $x = 2t^5 - 4t^4$.
   Find the:
   (i) Velocity after 4 seconds
   (ii) Acceleration after 4 seconds

   (8 marks)

5. A boy measures the length of a square object as 5.8 cm instead of 6 cm. He uses his measurement to calculate the area of the object. Find the percentage error in the:

   (a) Length of the side
   (b) Area of the square

   Correct your answers to 2 decimal places. **(8 marks)**

PART II

6. (a) The 12th term of a Geometric Progression is 128 times the 5th term and the sum of the 2nd and 6th terms is 102. Find the:

(i) Common ratio
(ii) 1st term
(iii) Difference between the 9th and 4th terms

(b) If $y = (3x^2 - 9)^3$, find $\frac{dy}{dx}$ at $x = 2$. **(12 marks)**

7. Two ships M and N take off from a port T at the same time. M moves on a bearing of S25°W while N moves on a bearing of N60°E. If both travel a distance of 20 km each from T, determine the:

(a) Distance between M and N, correct to 1 decimal place
(b) Bearing of N from M **(12 marks)**

8. (a) In an Arithmetic Progression (A.P.), the difference between the 14th and 7th term is 56 and the 12th term is $2\frac{1}{3}$ of the 4th term. Find the:

(i) Common difference
(ii) Sum of the first nine terms of the A.P.

(b) Evaluate $\int_{-4}^{1}(\frac{1}{2} - 2x)dx$.

(12 marks)

9. (a) Find the area of the region bounded by the curve $y = x^2 + 1$, the ordinates $x = 1$, $x = 2$ and the x-axis.

(b) (i) Find the inverse of the matrix $A = \begin{bmatrix} 4 & 2 \\ 5 & 1 \end{bmatrix}$.

(ii) Evaluate $A^{-1} \times A$.

(c) Given that $Q = \begin{pmatrix} 2 & 1 & 3 \\ 3 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix}$, find $|Q|$. **(12 marks)**

10. (a) Using a ruler and a pair of compasses only, construct:

(i) A triangle ABC in which $|AB| = 8$cm, $\angle CAB = 60°$ and $\angle ABC = 75°$
(ii) The locus l_1 of points equidistant from A and B
(iii) The locus l_2 of points 6cm from A

(b) (i) Measure $|H_1H_2|$, where H_1 and H_2 are the points of intersection of l_1 and l_2.

(ii) Measure $|AC|$. **(12 marks)**

11. (a) Use logarithm table to evaluate $\frac{53.76 \times 0.536}{(0.124)^3 \times 0.325}$.

(b) Solve the equation $\frac{3}{(x+2)} - \frac{1}{2(x-3)} = \frac{5}{x-3}$.

(12 marks)

12. **Table 2.3:** Frequency distribution

Class interval		45-49	50-54	55-59	60-64	65-69	70-74	75-79	80-84
Frequency		5	8	17	38	59	44	25	4

(a) Draw the ogive for **Table 2.3.**
(b) Use the ogive to estimate the:

(i) Median
(ii) Lower and upper quartiles
(iii) Semi – interquartile range

(12 marks)

QUESTION 1

Given:

$$x = \frac{2a+3}{3a-2}$$

$$\frac{x-1}{2x+1} = \left[\frac{2a+3}{3a-2} - 1\right] \div \left[2\left(\frac{2a+3}{3a-2}\right) + 1\right]$$

$$= \left[\frac{(2a+3)-(3a-2)}{3a-2}\right] \div \left[\frac{(4a+6)+(3a-2)}{3a-2}\right]$$

$$= \left[\frac{2a+3-3a+2}{3a-2}\right] \times \frac{(3a-2)}{4a+6+3a-2}$$

$$= \frac{5-a}{7a+4}$$

Translation from

QUESTION 2

Ratio of volume = (ratio of radius)^3

$$\frac{300}{V_2} = \left(\frac{r}{r/3}\right)^3$$

$$\frac{300}{V_2} = (3)^3$$

$$V_2 = \frac{300}{3^3} = \frac{300}{27} = 11.1 \text{ cm}^3$$

QUESTION 3

(a) $\frac{8}{3r^2-48} - \frac{4}{3r^2-12r}$

$= \frac{8}{3(r^2-16)} - \frac{4}{3r(r-4)}$

$= \frac{8}{3(r+4)(r-4)} - \frac{4}{3r(r-4)}$

$= \frac{8r - 4(r+4)}{3r(r+4)(r-4)}$

$= \frac{8r-4r-16}{3r(r+4)(r-4)}$

$= \frac{4r-16}{3r(r+4)(r-4)}$

$= \frac{4(r-4)}{3r(r+4)(r-4)}$

$= \frac{4}{3r(r+4)}$

(b) $\int (3x^2+2)^2 dx = \int (3x^2+2)(3x^2+2)\,dx$

$= \int (9x^4+12x^2+4)\,dx$

$= \frac{9x^5}{5} + \frac{12x^3}{3} + 4x + C$

$= \frac{9x^5}{5} + 4x^3 + 4x + C$

QUESTION 4

Given: $x = 2t^5 - 4t^4$

(i) Velocity, $v = \frac{dx}{dt} = 2(5)t^4 - 4(4)t^3$

$= 10t^4 - 16t^3$

After 4 secs,

$v = 10(4)^4 - 16(4)^3$

$v = 10(256) - 16(64)$

$v = 2560 - 1024$

$v = 1536 \text{ m/s}$

(ii) Acceleration, $a = \frac{dv}{dt} = \frac{d}{dt}(10t^4 - 16t^3)$

$a = 10(4)t^3 - 16(3)t^2$

$a = 40t^3 - 48t^2$

After 4 secs, $a = 40(4)^3 - 48(4)^2$

$= 2560 - 768$

$= 1792 \text{ m/s}^2$

QUESTION 5

(a) percentage error in length $= \frac{\text{Error}}{\text{True value}} \times 100\%$

$= \frac{|5.8-6|}{6} \times 100\%$

$= \frac{0.2}{6} \times 100\%$

$= 3.33\%$

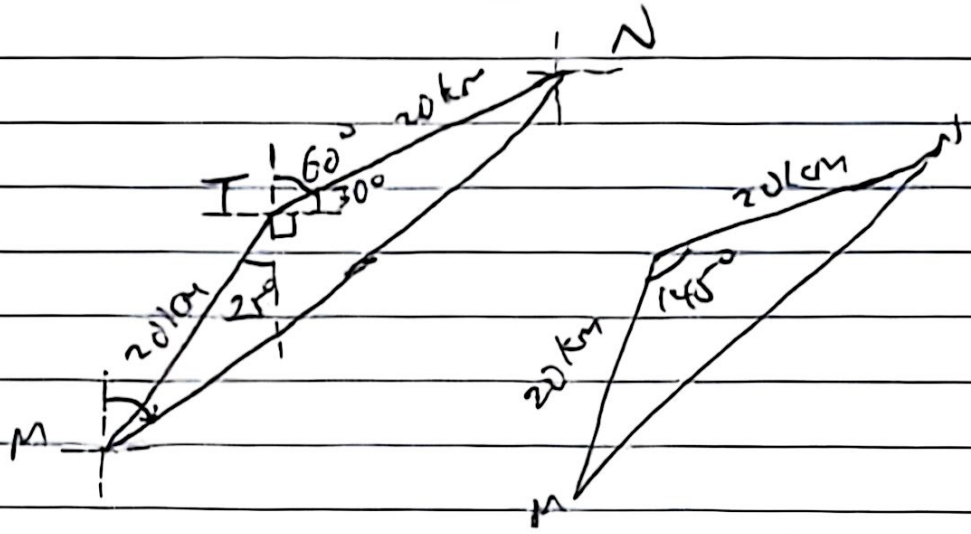
(b) percentage error in the area $= \frac{|5.8^2-6^2|}{6^2} \times 100\%$

$= \frac{|33.64-36|}{36} \times 100\%$

$= \frac{2.36}{36} \times 100\%$

$= 6.56\%$

QUESTION 7



(a) $\overline{MN}$ =

$$\sqrt{20^2+20^2-2(20)(20)\cos 145^\circ}$$

$$\overline{MN} = \sqrt{400+400-800(-\cos 35^\circ)}$$

$$= \sqrt{800+800(0.8192)}$$

$$= \sqrt{1455.36}$$

$$\overline{MN} = 38.15$$

$$\overline{MN} = 38.21\text{cm}$$

(b) Using sine rule: $\frac{20}{\sin M} = \frac{38.15}{\sin 145}$

$$\sin M = \frac{20\sin 145^\circ}{38.15}$$

$$\sin M = 0.3007$$

$$M = \sin^{-1}(0.3007)$$

$$M = 17.5^\circ$$

Bearing of N from M $= 25^\circ + 17.5^\circ$

$$= 42.5^\circ$$

QUESTION 8

(a) For A.P. $T_n = a + (n-1)d$

(i) $T_{14} - T_7 = 56$

$a + 13d - a - 6d = 56$

$7d = 56$

$d = 56/7 = 8$

Common difference $= 8$

(ii) $T_{12} = 2\frac{1}{3} \times T_4$

$a + 11d = \frac{7}{3}(a + 3d)$

$a + 11(8) = \frac{7}{3}(a + 3(8))$

$a + 88 = \frac{7a}{3} + 56$

$\frac{7a}{3} - a = 88 - 56$

$\frac{4a}{3} = 32$

$a = \frac{3}{4} \times 32 = 24$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_9 = \frac{9}{2}[2(24) + 8(8)]$

$= \frac{9}{2}[48 + 64]$

$= 504$

(b) $\int_{-4}^{1} \left(\frac{1}{2} - 2x\right) dx$

$= \left[\frac{1}{2}x - x^2\right]_{-4}^{1}$

$= \left[\frac{1}{2} - 1\right] - \left[\frac{1}{2}(-4) - (-4)^2\right]$

$= \left(-\frac{1}{2}\right) - (-2 - 16)$

$= -\frac{1}{2} + 18$

$= 17\frac{1}{2}$

QUESTION 9

(a) Area $= \int_1^2 (3x^2+1)\,dx = \left[\frac{x^3}{3} + x\right]_1^2$

$= \left[\frac{2^3}{3} + 2\right] - \left[\frac{1^3}{3} + 1\right]$

$= \left[\frac{8}{3} + 2\right] - \left[\frac{1}{3} + 1\right]$

$= \frac{8}{3} - \frac{1}{3} + 2 - 1$

$= \frac{7}{3} + 1 = 3\frac{1}{3}$ square units

(b) (i) Given: $A = \begin{bmatrix} 4 & 2 \\ 5 & 1 \end{bmatrix}$

$\text{Det}(A) = 4(1) - 5(2) = 4 - 10 = -6$

Cofactors of $A = \begin{bmatrix} 1 & -5 \\ -2 & 4 \end{bmatrix}$

Adjunct of $A = \begin{bmatrix} 1 & -2 \\ -5 & 4 \end{bmatrix}$

Inverse of $A = \frac{\text{Adjunct of } A}{\text{Det } A} = \frac{\begin{bmatrix} 1 & -2 \\ -5 & 4 \end{bmatrix}}{-6}$

$A^{-1} = \begin{bmatrix} -1/6 & 1/3 \\ 5/6 & -2/3 \end{bmatrix}$

(ii) $A^{-1} \times A = \begin{bmatrix} -1/6 & 1/3 \\ 5/6 & -2/3 \end{bmatrix} \begin{bmatrix} 4 & 2 \\ 5 & 1 \end{bmatrix}$

$= \begin{bmatrix} (-1/6 \times 4) + (1/3 \times 5) & (-1/6 \times 2) + (1/3 \times 1) \\ (5/6 \times 4) + (-2/3 \times 5) & (5/6 \times 2) + (-2/3 \times 1) \end{bmatrix}$

$= \begin{bmatrix} -2/3 + 5/3 & -1/3 + 1/3 \\ 10/3 - 11/3 & 5/3 - 2/3 \end{bmatrix}$

$= \begin{bmatrix} 3/3 & 0 \\ 0 & 3/3 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

QUESTION 6

(a) For a G.P. : $T_n = ar^{n-1}$

(i) $T_{12} = 128\,T_5$

$ar^{11} = 128\,ar^4$

$\frac{r^{11}}{r^4} = 128$;

$r^7 = 128 = 2^7$

$\Rightarrow r = 2$

Common ratio, $r = 2$

(ii) $T_2 + T_6 = 102$

$ar + ar^5 = 102$

$a = \frac{102}{(r + r^5)} = \frac{102}{2+2^5} = \frac{102}{2+32} = \frac{102}{34}$

First term, $a = 3$

(iii) $T_9 - T_4 = ar^8 - ar^3$

$= a(r^8 - r^3)$

$= 3(2^8 - 2^3)$

$= 3(256 - 8)$

$= 3(248)$

$= 744$

(b) Given: $y = (3x^2 - 9)^3$

let $u = 3x^2 - 9$; i.e $y = u^3$

$\Rightarrow du/dx = 6x$

$\Rightarrow \frac{dy}{du} = 3u^2$

$dy/dx = \frac{dy}{du} \times du/dx = 3u^2 \times 6x$

$= 3(3x^2 - 9)^2 \times 6x$

$dy/dx = 18x(3x^2 - 9)^2$

At $x = 2$

$dy/dx = 18(2)\left[3(2^2) - 9\right]^2$

$= 36\,[3]^2$

$= 36 \times 9$

$= 324$