

PAPER II

PART I

Answer all questions in this part.

1. Solve the equation $2^{2x} - 3(2^{x+2}) + 32 = 0$. (8 marks)
2. If $x = \frac{2a+3}{3a-2}$, express $\frac{x-1}{2x+1}$ in terms of a . (8 marks)
3. (a) Find the distance between the points A(-3, 2) and B(4, -6).
(b) Find the acute angle between the lines $2x + 3y = 5$ and $3x + 2y = 1$ correct to one decimal place. (8 marks)
4. (a) Out of 930 students in a college, 450 are girls. $\frac{2}{3}$ of the girls and $\frac{3}{4}$ of the boys are less than 20 years old. If a student is picked at random, find the probability that the student is less than 20 years.
(b) There are 6 red, 4 blue and 8 green balls in a bag. Two balls are picked at random one after the other without replacement. Find the probability that the two balls are either red or blue. (8 marks)
5. (a) If $y = \left[\frac{1}{3}x^3 + 2x + 5 \right]^2$, find $\frac{dy}{dx}$.
(b) Given that $\log 3 = 0.4771$ and $\log 2 = 0.3010$, evaluate $\log 162 + \log 36 - \log 54$. (8 marks)

PART II

Answer five questions only in this part.

6. (a) Express the sum of 112012_{three} and 21121_{three} in base six.

(b) If $\frac{8^{\frac{x}{6}} \times 16^{\frac{x}{12}}}{2^{\frac{x}{8}} \times 4^{\frac{x}{4}}} = 10^0$, find the value of x .

(12 marks)

7. (a) The 12th term of a Geometric Progression is 128 times the 5th term and the sum of the 2nd and 6th terms is 102. Find the:

- (i) Common ratio
(ii) 1st term
(iii) Difference between the 9th and 4th terms

(b) If $y = (3x^2 - 9)^3$, find $\frac{dy}{dx}$ at $x = 2$.

(12 marks)

8. (a) Find the equation whose roots are $\frac{5}{6}$ and $-\frac{3}{4}$.

(b) If α, α^2 are the roots of the equation $64x^2 + px + 27 = 0$, find the value of p .

(c) Using completing the square method, solve the equation $25x^2 - 25x + 6 = 0$.

(12 marks)

9. In Fig. 2.1, OAB is a sector of a circle with centre O and radius 16 cm.

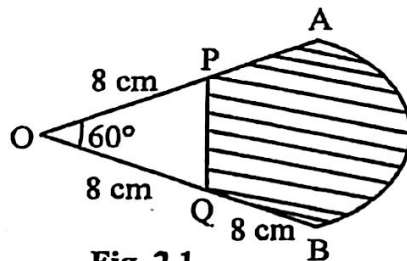


Fig. 2.1

- (a) Find the area of the shaded portion.
(b) Calculate the perimeter of the shaded portion.
(Take $\pi = \frac{22}{7}$)

(12 marks)

10. (a) Draw the graph of $y = x^2 - 3x + 2$ for $-2 \leq x \leq 4$, using a scale of 2 cm to 1 unit on x -axis and 2 cm to 2 units on y -axis.
- (b) From your graph, determine the:
- (i) Equation of the line of symmetry of the curve
 - (ii) Roots of the equation $x^2 - 3x + 2 = 0$
 - (iii) Gradient of the curve at $x = 0$
 - (iv) Minimum value of y
- (12 marks)

11. An aeroplane flies from a town P (Lat. 10°N , Long. 40°E) to another town Q (Lat. 10°N , Long. 20°W). It later flies to a third town R (Lat. 32°N , Long. 20°W). Calculate, correct to 3 significant figures, the:

- (a) Distance between P and Q along their parallel of latitude
- (b) Distance between Q and R along their lines of longitude
- (c) Total time taken to travel from P to R if it travels at a speed of 3,552 km/h

(Take $R = 6400 \text{ km}$, $\pi = \frac{22}{7}$)

(12 marks)

12. The length of 50 iron rods to the nearest cm are as follows:

13	15	22	27	41	45	54	20	23	11
31	23	35	13	45	18	39	23	12	27
54	26	60	36	53	21	42	31	25	35
16	37	27	23	55	33	41	26	58	39
18	28	35	12	49	25	43	16	11	22

- (a) Prepare a frequency distribution table, using class intervals of 11-20, 21-30, 31-40, ...
 - (b) Using an assumed mean of 35.5, compute the standard deviation.
 - (c) Find the:
 - i. Range of the distribution
 - ii. Variance
- (12 marks)

PAPER III
Answer all questions.

Each question is followed by five options lettered A to E. Choose the correct option for each question and shade in pencil on your answer sheet the answer space that bears the same letter as the option you have chosen. Give only one answer to each question and erase completely any answer you wish to change. Do all rough work on this question paper.

An example is given below:

The product of three numbers is 3876. Two of the numbers are 17 and 19. What is the third number?

- A. 2
- B. 3
- C. 6
- D. 12
- E. 57

The correct option is '12' which is lettered B. Therefore, answer space B would be shaded as shown below:

- | [A] | [B] | [C] | [D] | [E] |
|---|---|-----|-----|-----|
| <p>1. If $J \propto (3 + \frac{Q}{2})$, which of the following statement is true? J</p> <ul style="list-style-type: none"> A. varies directly as 3 and inversely as Q. B. varies inversely as Q. C. varies directly as Q. D. varies jointly as 3 and directly as Q. E. is partly constant and partly varies as Q. | <p>2. A fair die is rolled once. What is the probability of obtaining a 3 or 4?</p> <ul style="list-style-type: none"> A. $\frac{5}{6}$ B. $\frac{2}{3}$ C. $\frac{1}{2}$ D. $\frac{1}{3}$ E. $\frac{3}{5}$ | | | |

10. Express $\frac{2p-3q}{15} + \frac{p-2q}{15}$ as a single fraction.

- A. $\frac{p+1}{15}$
 B. $\frac{7p}{30}$
 C. $\frac{p}{5}$
 D. $4p$
 E. $2p+1$

11. Simplify $\frac{2x+1}{6x} + \frac{4y-3}{2y}$.

- A. $\frac{10xy-9x+y}{6xy}$
 B. $\frac{15xy-9x+y}{6xy}$
 C. $\frac{14xy-9x-y}{6xy}$
 D. $\frac{14xy-9x+y}{6xy}$
 E. $\frac{14xy+9x+y}{6xy}$

12. Two fair dice are tossed together once. Find the probability that the sum of the outcome is at least 10.

- A. $\frac{1}{6}$
 B. $\frac{1}{7}$
 C. $\frac{1}{5}$
 D. $\frac{1}{9}$
 E. $\frac{1}{8}$

13. Consider the truth table below:

P	Q	Y
T	T	T
T	F	F
F	T	F
F	F	F

Which of the following represents Y?

- A. $\sim(P \vee Q)$
 B. $P \wedge Q$
 C. $\sim(P \wedge Q)$
 D. $P \vee Q$
 E. $P \Rightarrow Q$

14. An aircraft flies from Q (25°N , 38°E) to R (72°S , 38°E). Calculate the distance QR along the line of longitude, correct to the nearest kilometre.

(Take $R = 6,400 \text{ km}$, $\pi = \frac{22}{7}$)

- A. 3,930
 B. 9,810
 C. 10,839
 D. 1,039
 E. 29,839

15. Express the product of 0.016 and 3.54 in standard form.

- A. 5.664×10^2
 B. 5.664×10^{-3}
 C. 5.664×10^3
 D. 5.664×10^{-3}
 E. 5.664×10^{-1}

23. Find the gradient of the curve
 $y = 2x^2 + 3x + 4$ at the point $x = 2$.

A. 13
 B. 0
 C. 5
 D. 13
 E. 5

24. The table below shows the scores obtained by students in a Mathematics test.

Scores of students

Scores	1	2	3	4	5	6	7	8	9	10
Frequency	3	5	3	7	5	1	5	6	3	2

What percentage of the students scored at least 6 marks?

A. 41.5
 B. 44.5
 C. 42.3
 D. 40.5
 E. 42.5

25. Solve the equation $x = \frac{5}{3x+2}$.

A. $-\frac{5}{3}$ or 1
 B. -5 or 3
 C. $-\frac{2}{3}$ or 1
 D. 3 or 1
 E. $-\frac{1}{3}$ or 2

26. Given that $X = \begin{pmatrix} 4 & -11 \\ -8 & 5 \end{pmatrix}$ and $Y = \begin{pmatrix} -5 & 2 \\ 0 & 6 \end{pmatrix}$, evaluate $4X - 3Y$.

A. $\begin{pmatrix} 31 & -50 \\ -32 & 2 \end{pmatrix}$
 B. $\begin{pmatrix} 14 & 50 \\ -32 & 38 \end{pmatrix}$
 C. $\begin{pmatrix} 31 & -38 \\ -32 & 2 \end{pmatrix}$
 D. $\begin{pmatrix} 41 & -38 \\ -32 & 2 \end{pmatrix}$
 E. $\begin{pmatrix} 31 & 50 \\ 32 & 38 \end{pmatrix}$

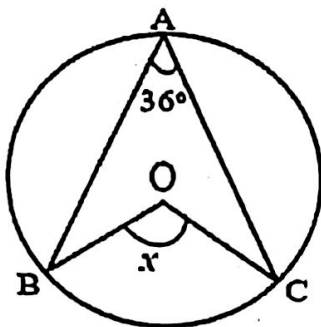
27. If y is a whole number such that $2y + 1 = 4 \pmod{7}$, find the least value of y .

A. 5
 B. 2
 C. 1
 D. 4
 E. 3

28. Simplify $\frac{x^2 - 9}{x^2 + 2x - 15}$.

A. $\frac{x-3}{x+5}$
 B. $\frac{x+3}{x+5}$
 C. $\frac{2x+3}{2x+5}$
 D. $\frac{x+3}{x+10}$
 E. $\frac{x+3}{x-5}$

5. Find the value of x in figure below where O is the centre of the circle.



- A. 38°
 B. 48°
 C. 44°
 D. 18°
 E. 72°
6. What must be added to the expression $x^2 + \frac{8x}{3}$ to make it a perfect square?
- A. $\frac{64}{9}$
 B. $\frac{4}{93}$
 C. $\frac{8}{3}$
 D. $\frac{16}{3}$
 E. $\frac{16}{9}$

7. Use the information below to answer the question.

The table below shows the scores obtained by 20 students in a Mathematics test.

Scores of students

Scores	8	10	12	18	20
No. of students	3	5	6	2	1

Find the median score.

- A. 12
 B. 10
 C. 8
 D. 20
 E. 15
8. Simplify $32^{-3/5}$.
- A. 16
 B. $\frac{1}{4}$
 C. $\frac{1}{8}$
 D. 8
 E. 4
9. If $\left| \frac{4}{5} \frac{x}{3} \right| = 42$, find the value of x .
- A. -4
 B. -6
 C. 11
 D. 8
 E. 6

57. If the mean of the numbers 2, 5, $(x + 2)$, $(x + 1)$, 7 and 9 is 6, find the value of x .

- A. 7
- B. 5
- C. 2
- D. 9
- E. 4

58. A chord of a circle subtends an angle of 120° at the centre with radius 6 cm. Find the area of the minor segment, correct to three significant figures.

(Take $\pi = \frac{22}{7}$)

- A. 22.1 cm^2
- B. 113.0 cm^2
- C. 15.6 cm^2
- D. 53.3 cm^2
- E. 37.7 cm^2

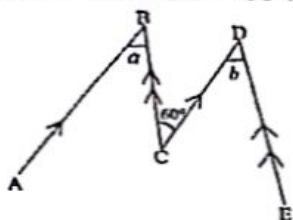
59. Find the equation of a straight line passing through the points M(3, 3) and N(2, 1).

- A. $y = 4x - 9$
- B. $y = 4x + 9$
- C. $y = 4x + 11$
- D. $y = 4x - 11$
- E. $y = -4x - 9$

60. If $\frac{5}{2}$ and $-\frac{2}{3}$ are the roots of the equation $ax^2 + bx + c = 0$, evaluate $(a - c + b)$.

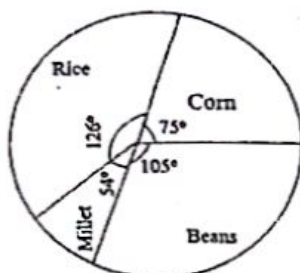
- A. 9
- B. 12
- C. 8
- D. 10
- E. 5

36. Find the values of a and b in the figure below, given that $AB \parallel CD$, $CB \parallel ED$ and $\angle BCD = 60^\circ$.



- A. $a = 50^\circ, b = 50^\circ$
 B. $a = 60^\circ, b = 60^\circ$
 C. $a = 40^\circ, b = 40^\circ$
 D. $a = 30^\circ, b = 40^\circ$
 E. $a = 20^\circ, b = 30^\circ$
37. An agent received 25% commission for selling a house for ₦2,600,000.00. What amount did he receive?
- A. ₦750,000.00
 B. ₦500,000.00
 C. ₦650,000.00
 D. ₦250,000.00
 E. ₦700,000.00
38. The surface area of a sphere is 704 cm^2 . Find the volume of the sphere, correct to the nearest whole number.
- A. 1030 cm^3
 B. 1757 cm^3
 C. 1412 cm^3
 D. 980 cm^3
 E. 1680 cm^3
39. A man is four times as old as his son. If the difference between their ages is 36, find the sum of their ages.
- A. 40
 B. 60
 C. 45
 D. 144
 E. 65
40. Find the minimum value of the curve $y = 3x^2 - 3x$.
- A. $2\frac{1}{4}$
 B. $\frac{3}{4}$
 C. $\frac{1}{2}$
 D. $-\frac{3}{4}$
 E. $-\frac{1}{2}$
41. A man deposited a sum of ₦45,000.00 in a bank at 3% simple interest. Calculate the amount at the end of 7 years.
- A. ₦64,450.00
 B. ₦44,450.00
 C. ₦54,450.00
 D. ₦63,900.00
 E. ₦35,550.00
42. Differentiate $6x^4 + 5x^3 + 2x$ with respect to x .
- A. $6x^3 + 5x^2 + 2$
 B. $6x^5 + 5x^4 + 2x^2$
 C. $24x^3 + 5x^2 + 2$
 D. $24x^5 + 5x^4 + 2x^2$
 E. $24x^3 + 15x^2 + 2$

50. The figure below shows the distribution of crops harvested in a farm. The total bags harvested for all the crops is 300.



What percentage of the crops harvested is beans?

- A. 33.4
B. 29.2
C. 15.1
D. 18.0
E. 25.0
51. Calculate the variance of the data: 6, 4, 5, 9.
- A. 3.5
B. 6.0
C. 3.4
D. 1.8
E. 2.4
52. In a class of 60 students, 45 study Biology and 35 study Geography. If 8 study none of the two subjects, how many students study both subjects?

- A. 10
B. 28
C. 18
D. 22
E. 48

53. Find the range of values of x for which $4 + x \leq 7$ and $13 + x \geq 8$ are both satisfied.

- A. $-3 \leq x \leq 5$
B. $5 \leq x < 3$
C. $-5 \leq x \geq 3$
D. $-5 \leq x \leq 3$
E. $-3 \leq x \geq 5$

54. Find the equation whose roots are $2\frac{1}{2}$ and $1\frac{1}{2}$.

- A. $4x^2 + 8x + 15 = 0$
B. $4x^2 - 16x + 15 = 0$
C. $2x^2 - 8x + 15 = 0$
D. $8x^2 - 10x + 15 = 0$
E. $x^2 - 4x + 15 = 0$

55. The bearing of two points Q and R from a point P are 030° and 120° respectively. If $|PQ| = 12$ m and $|PR| = 5$ m, find $|QR|$.

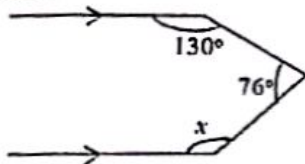
- A. 9 m
B. 7 m
C. 11 m
D. 5 m
E. 13 m

56. If $\begin{vmatrix} 2 & 3 & -1 \\ 5 & x & 3 \\ -3 & 5 & 2 \end{vmatrix} = -106$,

find the value of x .

- A. 5
B. 4
C. 6
D. 6
E. 5

43. Find the value of x in the figure below.



- A. 106°
 B. 116°
 C. 130°
 D. 126°
 E. 154°

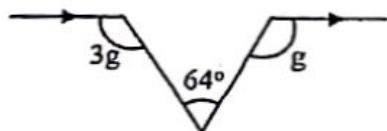
44. Find the range of the following numbers: 7, 2, 8, 6, 11, 9, 4, 5, 2, 3.

- A. 9
 B. 5
 C. 2
 D. 6
 E. 11

45. A sequence is given by $x + y$, $2x + 3y$, $3x + 5y$, ... If the n th term is $9x + 17y$, find n .

- A. 8
 B. 13
 C. 6
 D. 19
 E. 9

46. Calculate the value of g in the figure below.



- A. 112°
 B. 34°
 C. 224°
 D. 61°
 E. 64°

47. Solve for x , if $\sqrt{\frac{x+2}{x-1}} = 4^{-\frac{1}{2}}$.

- A. -3
 B. -2
 C. -1
 D. 0
 E. 1

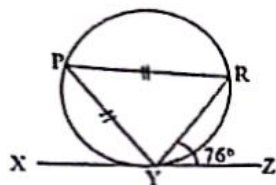
48. Express $\frac{4x}{3-x} + \frac{3x}{x-2}$ as a single fraction.

- A. $\frac{x(x-17)}{(3-x)(x+2)}$
 B. $\frac{17x}{(3-x)(x+2)}$
 C. $\frac{x(x+17)}{(3-x)(x+2)}$
 D. $\frac{x(x+17)}{(x+3)(x+2)}$
 E. $\frac{x(x+17)}{(x-3)(x+2)}$

49. Find the sum and the product of the roots of the equation $3x^2 - 2x - 6 = 0$ respectively.

- A. $-2, \frac{2}{3}$
 B. $-2, -\frac{2}{3}$
 C. $\frac{2}{3}, -2$
 D. $-\frac{1}{3}, -2$
 E. $-2, -\frac{1}{3}$

29. In the figure below, XYZ is a tangent to the circle at Y, $|PR| = |PY|$ and $\angle RYZ = 76^\circ$. Calculate $\angle PRY$.



- A. 104°
 B. 46°
 C. 126°
 D. 52°
 E. 76°
30. Solve the equation $\log_x 2 + \log_x 8 = 4$.
- A. 8
 B. 16
 C. 2
 D. 4
 E. 12
31. The roots of the equation $2x^2 - 10x + 8 = 0$ are α and β . Calculate $\alpha^2 + \beta^2$.
- A. 24
 B. 33
 C. 12
 D. 28
 E. 17
32. The roots of the equation $2x^2 - 10x + 8 = 0$ are α and β . Calculate $\frac{1}{\alpha} + \frac{1}{\beta}$.
- A. $\frac{1}{2}$
 B. $\frac{3}{2}$
 C. $\frac{1}{4}$
 D. $\frac{3}{4}$
 E. $\frac{5}{4}$
33. Expand $(2x - 1)(x + 2)$.
- A. $2x^2 - 3x - 2$
 B. $2x^2 + 5x - 2$
 C. $2x^2 - 5x + 2$
 D. $2x^2 + 3x + 2$
 E. $2x^2 + 3x - 2$
34. If 15 is the mean of the following set of numbers: 4, 6, x, 10, 5, find the value of x.
- A. 50
 B. 75
 C. 42
 D. 55
 E. 65
35. Calculate the amount, if a sum of ₦320,000.00 is invested for 4 years at 5% per annum simple interest.
- A. ₦384,000.00
 B. ₦326,400.00
 C. ₦448,000.00
 D. ₦584,000.00
 E. ₦648,000.00

16. If p varies inversely as the square root of $(m + 2)$ and $p = 12$ when $m = 7$, find m when $p = 6$.

A. 18
B. 6
C. 36
D. 9
E. 34

17. Evaluate $\frac{2x^2yz}{2y - z}$, when $x = 3$, $y = -4$ and $z = -5$.

A. 360
B. -360
C. 120
D. -120
E. -300

18. Find the value of x if $58 \equiv x \pmod{8}$.

A. 5
B. 2
C. 0
D. 7
E. 3

19. The base of a triangle is 3 cm longer than its height. If the area is 44cm^2 , find the length of the base.

A. 12 cm
B. 8 cm
C. 9 cm
D. 10 cm
E. 11 cm

20. Solve the equation $16^x = 2^{\frac{1}{2}} \times 8$.

A. $\frac{2}{7}$
B. $\frac{1}{7}$
C. $\frac{9}{11}$
D. $\frac{8}{9}$
E. $\frac{7}{8}$

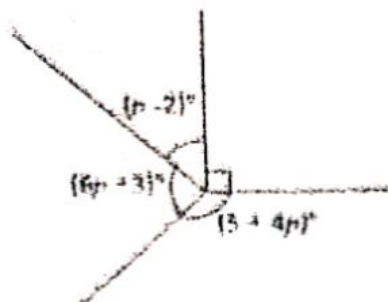
21. If N varies directly as M and $N = 8$ when $M = 20$, find the value of M when $N = 7$.

A. 18.5
B. 13.4
C. 17.5
D. 15.4
E. 11.5

22. A bag contains 3 green and 2 brown identical balls. If 2 balls are picked at random from the bag one after the other without replacement, find the probability that they are of the same colour.

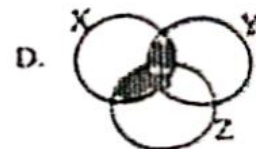
A. $\frac{1}{2}$
B. $\frac{2}{85}$
C. $\frac{1}{10}$
D. $\frac{3}{10}$
E. $\frac{1}{4}$

3. Find the value of p in the figure below:



- A. 96°
 B. 88°
 C. 26°
 D. 48°
 E. 24°

4. Which of the following describes $X \cup (Y \cap Z)$?



TYPE C

1-10: EDEEEEEACBC

11-20: DABCBEDBEE

21-30: C--BABEBDC

31-40: EEEAABCBBBD

41-50: CEEAEDACCB

51-60: ABDBECBA-E

Edited 9:01 AM ✓✓

QUESTION

$$2^{2x} - 3(2^{x+2}) + 32 = 0$$

$$(2^x)^2 - 3(2^x \times 2^2) + 32 = 0$$

$$(2^x)^2 - 12(2^x) + 32 = 0$$

$$(2^x)^2 - 8(2^x) - 4(2^x) + 32 = 0$$

$$2^x [2^x - 8] - 4[2^x - 8] = 0$$

$$(2^x - 4)(2^x - 8) = 0$$

$$2^x - 4 = 0 \quad \text{OR} \quad 2^x - 8 = 0$$

$$2^x = 4$$

$$\text{OR} \quad 2^x = 8$$

$$2^x = 2^2$$

$$\text{OR} \quad 2^x = 2^3$$

$$x = 2$$

$$\text{OR} \quad x = 3$$

complete solution given

QUESTION 2

Given:

$$x = \frac{2a+3}{3a-2}$$

$$\frac{x-1}{2x+1} = \left[\frac{2a+3}{3a-2} - 1 \right] \div \left[2\left(\frac{2a+3}{3a-2}\right) + 1 \right]$$

$$= \left[\frac{(2a+3) - (3a-2)}{3a-2} \right] \div \left[\frac{(4a+6) + (3a-2)}{3a-2} \right]$$

$$= \left[\frac{2a+3-3a+2}{3a-2} \right] \times \frac{(3a-2)}{4a+6+3a-2}$$

$$= \frac{5-a}{7a+4}$$

Question 3

(a) Given $A(-3, 2)$ and $B(4, -6)$

$$AB = \sqrt{(-6-2)^2 + (4-(-3))^2}$$

$$= \sqrt{(-8)^2 + (7)^2}$$

$$= \sqrt{64 + 49}$$

$$= \sqrt{113}$$

(b) Given $2x + 3y = 5 \Rightarrow 3y = -2x + 5$

$$y = -\frac{2}{3}x + \frac{5}{3}$$

$$\text{gradient, } m_1 = -\frac{2}{3}$$

Also: $3x + 2y = 1$

$$2y = -3x + 1$$

$$y = -\frac{3}{2}x + \frac{1}{2}$$

$$\text{gradient, } m_2 = -\frac{3}{2}$$

$$\tan \theta = \frac{m_1 - m_2}{1 + m_1 m_2} = \frac{-\frac{2}{3} - (-\frac{3}{2})}{1 + (-\frac{2}{3})(-\frac{3}{2})}$$

$$\tan \theta = \frac{\frac{5}{6}}{2} = \frac{5}{12}$$

$$\theta = \tan^{-1}\left(\frac{5}{12}\right) = 22.6^\circ$$

$$\text{Acute angle} = 22.6^\circ$$

QUESTION 4

(a) Total no of students = 930

No of girls = 450 and no of boys = $930 - 450 = 480$

$\frac{2}{3}$ of the girls = $\frac{2}{3} \times 450 = 300$

$\frac{3}{4}$ of boys = $\frac{3}{4} \times 480 = 360$

No of students less than 20 years = $300 + 360 = 660$

prob (< 20 years) = $\frac{660}{930} = \frac{22}{31}$

01/04/2019

(b) Total no of balls = $6 + 4 + 8 = 18$ balls

prob (red) = $\frac{6}{18}$; prob (blue) = $\frac{4}{18}$; prob (green) = $\frac{8}{18}$

prob (either red or blue) = [1st red & 2nd blue] + [1st blue & 2nd red]

= $\left[\frac{6}{18} \times \frac{4}{17} \right] + \left[\frac{4}{18} \times \frac{6}{17} \right]$

= $\left[\frac{1}{3} \times \frac{4}{17} \right] + \left[\frac{1}{3} \times \frac{4}{17} \right]$

= $\frac{4}{51} + \frac{4}{51}$

= $\frac{8}{51}$

QUESTION 5

④ $y = \left[\frac{1}{3}x^3 + 2x + 5 \right]^2$

Let $u = \frac{1}{3}x^3 + 2x + 5$

$\frac{du}{dx} = \frac{1}{3}(3)x^2 + 2 = x^2 + 2$

$y = u^2$

$\frac{dy}{du} = 2u$

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = 2u \times (x^2 + 2)$

$\frac{dy}{dx} = 2 \left[\frac{1}{3}x^3 + 2x + 5 \right] (x^2 + 2)$

$= 2(x^2 + 2) \left(\frac{1}{3}x^3 + 2x + 5 \right)$

⑤ $\log 162 + \log 36 - \log 54$

$= \log 162 - \log 54 + \log 36$

$= \log \left(\frac{162}{54} \right) + \log 36$

$= \log 3 + \log 36$

$= \log 3 + \log 6^2$

$= \log 3 + 2 \log 6$

$= \log 3 + 2 [\log 3 + \log 2]$

$= \log 3 + 2 \log 3 + 2 \log 2$

$= 3 \log 3 + 2 \log 2$

$= 3(0.4771) + 2(0.3010)$

$= 1.4313 + 0.602$

$= 2.0333$

QUESTION 6

(a)

$$\begin{array}{r}
 \textcircled{1} \textcircled{1} \quad \textcircled{1} \textcircled{1} \\
 1 \ 1 \ 2 \ 0 \ 1 \ 2_{\text{three}} \\
 + \quad 2 \ 1 \ 1 \ 2 \ 1_{\text{three}} \\
 \hline
 2 \ 1 \ 0 \ 2 \ 1 \ 0_{\text{three}}
 \end{array}$$

$$\begin{aligned}
 210210_{\text{three}} &= (2 \times 3^5) + (1 \times 3^4) + (0 \times 3^3) + (2 \times 3^2) + (1 \times 3^1) + (0 \times 3^0) \\
 &= 486 + 81 + 0 + 18 + 3 + 0 \\
 &= 588_{\text{ten}}
 \end{aligned}$$

$$\begin{array}{r}
 588_{\text{ten}} = 6 \overline{) 588} \\
 \underline{6 \ 98} \quad R \ 0 \\
 \underline{6 \ 16} \quad R \ 2 \\
 \underline{6 \ 2} \quad R \ 4 \\
 0 \quad R \ 2
 \end{array}$$

$\therefore \text{Sum} = 2420_{\text{six}}$

(b)

$$\begin{aligned}
 8^{2/6} \times 16^{2/12} &= 10^0 \\
 2^{3/8} \times 4^{3/4} &= 1
 \end{aligned}$$

$$\left[2^3 \right]^{2/6} \times \left[2^4 \right]^{2/12} = 1$$

$$2^{3/8} \times \left[2^2 \right]^{3/4}$$

$$2^{2/2} \times 2^{2/3} = 1$$

$$2^{3/8} \times 2^{3/2}$$

$$2^{2/2 + 3/3} = 1$$

$$2^{3/8 + 3/2}$$

$$\begin{aligned}
 2^{52/6} \\
 \underline{2^{15/8}} &= 1
 \end{aligned}$$

i.e

$$2^{5/6} = 2^{15/8}$$

$$\frac{5x}{6} = \frac{15}{8}$$

$$x = \frac{6}{5} \times \frac{15}{8}$$

$$x = \frac{18}{8}$$

$$x = \frac{9}{4}$$

$$x = 2\frac{1}{4}$$

Question 7

(a) For a G.P. : $T_n = ar^{n-1}$

(i) $T_{12} = 128 T_5$

$$ar^{11} = 128 ar^4$$

$$\frac{r^{11}}{r^4} = 128 ;$$

$$r^7 = 128 = 2^7$$

$$\Rightarrow r = 2$$

Common ratio, $r = 2$

(ii) $T_2 + T_6 = 102$

$$ar + ar^5 = 102$$

$$a = \frac{102}{(r+r^5)} = \frac{102}{2+2^5} = \frac{102}{2+32} = \frac{102}{34}$$

First term, $a = 3$

(iii) $T_9 - T_4 = ar^8 - ar^3$

$$= a(r^8 - r^3)$$

$$= 3(2^8 - 2^3)$$

$$= 3(256 - 8)$$

$$= 3(248)$$

$$= 744$$

(b) Given, $y = (3x^2 - 9)^3$

let $u = 3x^2 - 9$; i.e. $y = u^3$

$$\Rightarrow \frac{du}{dx} = 6x$$

$$\Rightarrow \frac{dy}{du} = 3u^2$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = 3u^2 \times 6x$$

$$= 3(3x^2 - 9)^2 \times 6x$$

$$\frac{dy}{dx} = 18x(3x^2 - 9)^2$$

At $x = 2$

$$\frac{dy}{dx} = 18(2)[3(2^2) - 9]^2$$

$$= 36[3]^2$$

$$= 36 \times 9$$

$$= 324$$

QUESTION 8

④ Equation: $x^2 - (\text{sum of roots})x + (\text{product of roots}) = 0$
 $x^2 - \left(\frac{5}{6} - \frac{3}{4}\right)x + \left(-\frac{3}{4} \times \frac{5}{6}\right) = 0$
 $x^2 - \left(\frac{10-9}{12}\right)x + \left(-\frac{5}{8}\right) = 0$
 $x^2 - \left(\frac{1}{12}\right)x - \frac{5}{8} = 0$

multiply through by 24

$$24x^2 - 2x - 15 = 0$$

⑤ Given: $64x^2 + px + 27 = 0$

divide through by 64

$$x^2 + \frac{p}{64}x + \frac{27}{64} = 0$$

sum of roots = $-\frac{p}{64} = \alpha + \alpha^2$ — (1)

product of roots = $\frac{27}{64} = \alpha(\alpha^2)$ — (2)

From eq(2) $\alpha^3 = \frac{27}{64}$

$$\alpha = \sqrt[3]{\frac{27}{64}} = \frac{3}{4}$$

From equ (1) $-\frac{p}{64} = \frac{3}{4} + \left(\frac{3}{4}\right)^2$

$$-\frac{p}{64} = \frac{3}{4} + \frac{9}{16}$$

multiply through by 64

$$-p = 16(3) + 4(9)$$

$$-p = 84$$

$$p = -84$$

(c) $25x^2 - 25x + 6 = 0$

$$x^2 - x + \frac{6}{25} = 0$$

$$x^2 - x = -\frac{6}{25}$$

$$\left(x - \frac{1}{2}\right)^2 - \frac{1}{4} = -\frac{6}{25}$$

$$\left(x - \frac{1}{2}\right)^2 = \frac{1}{4} - \frac{6}{25} = \frac{25-24}{100} = \frac{1}{100}$$

$$\left(x - \frac{1}{2}\right)^2 = \frac{1}{100}$$

$$x - \frac{1}{2} = \pm \frac{1}{10}$$

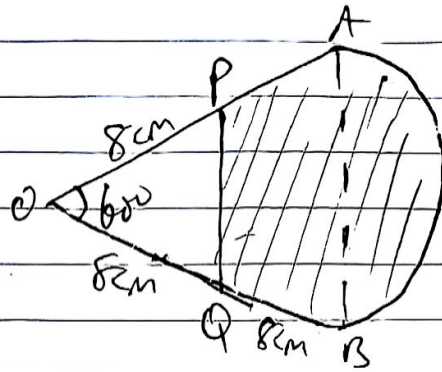
$$x = \frac{1}{2} \pm \frac{1}{10}$$

$$x = \frac{1}{2} + \frac{1}{10} \quad \text{or} \quad x = \frac{1}{2} - \frac{1}{10}$$

$$x = \frac{3}{5} \quad \text{or} \quad x = \frac{2}{5}$$

QUESTION 9

(a)



$$\text{Area of sector OAB} = \frac{\theta}{360} \times \pi r^2$$

$$= \frac{60}{360} \times \frac{22}{7} \times \frac{16}{1} \times \frac{16}{1}$$

$$= \frac{2816}{21} \text{ cm}^2$$

$$\text{Area of triangle OPQ} = \frac{1}{2} \sin \theta \times |OP| \times |OQ|$$

$$= \frac{1}{2} \times \sin 60^\circ \times 8^2$$

$$= 32 \times \frac{\sqrt{3}}{2} = 16\sqrt{3}$$

$$\text{Area of shaded portion} = \frac{2816}{21} - 16\sqrt{3}$$

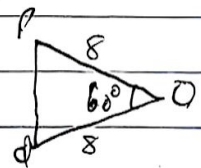
$$= \frac{2816 - 336\sqrt{3}}{21}$$

$$= 106.38 \text{ cm}^2$$

(b) Arc AB = $\frac{\theta}{360} \times 2\pi r$

$$= \frac{60}{360} \times 2 \times \frac{22}{7} \times \frac{16}{1}$$

$$= \frac{352}{21} \text{ cm}$$



$$\text{Perimeter of shaded portion} = 8 + 8 + \frac{352}{21} + \overline{PQ}$$

$$= \frac{688}{21} + \overline{PQ}$$

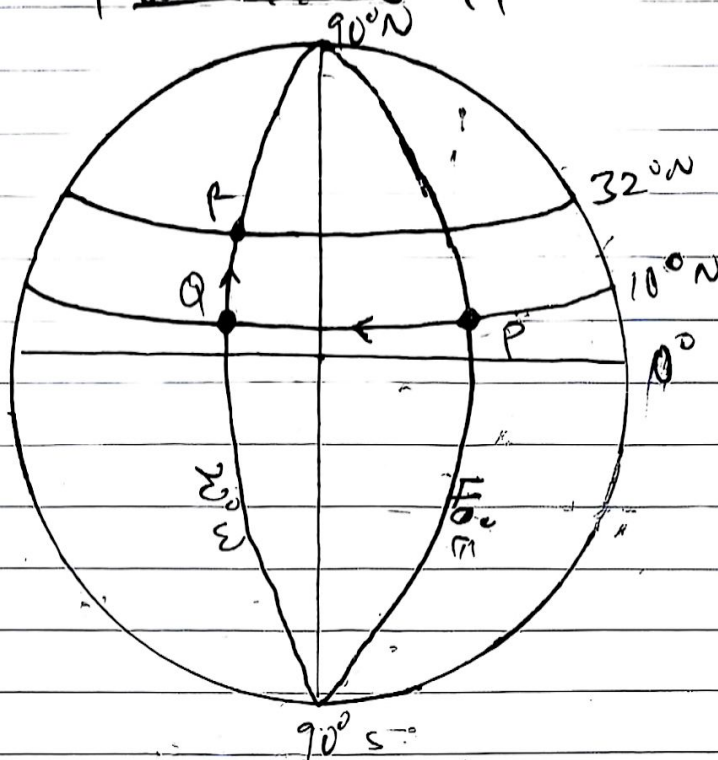
Using cosine! $\overline{PQ} = \sqrt{8^2 + 8^2 - 2(8)(8)\cos 60^\circ}$

$$= \sqrt{64}$$

$$\overline{PQ} = 8$$

$$\therefore \text{Perimeter} = \frac{688}{21} + 8 = \frac{856}{21} = 40.76 \text{ cm}$$

QUESTION 11



(a) $PQ = \frac{\theta_1}{360^\circ} \times 2\pi R \cos \phi$ where: $\phi = 10^\circ$
 $\theta_1 = 20 + 40 = 60^\circ$

$$= \frac{60}{360} \times 2 \times \frac{22}{7} \times 6400 \times \cos 10^\circ$$

$$= \frac{1}{3} \times \frac{22}{7} \times 6400 \times 0.9848$$

$$= 6602.9$$

$$= 6600 \text{ km (3 s.f.)}$$

(b) $QR = \frac{\theta_2}{360^\circ} \times 2\pi R$ where $\theta_2 = 32 - 10 = 22^\circ$

$$= \frac{22^\circ}{360^\circ} \times \frac{2}{1} \times \frac{22}{7} \times 6400$$

$$= 2458.4 \text{ km}$$

$$= 2460 \text{ km (3 s.f.)}$$

(c) Total time = $\frac{\text{Total distance}}{\text{Average speed}}$ = $\frac{6602.9 + 2458.4}{3552}$

$$= \frac{9061.3}{3552}$$

$$= 2.551 \text{ hours}$$

$$\approx 2.55 \text{ hours (3 s.f.)}$$

QUESTION 12

(a) ~~class interval~~

$$A = 35.5$$

class interval	frequency f	midpoint x	$d = x - A$	d^2	fd	fd^2
11-20	12	15.5	-20	400	-240	4800
21-30	15	25.5	-10	100	-150	1500
31-40	10	35.5	0	0	0	0
41-50	7	45.5	10	100	70	700
51-60	6	55.5	20	400	120	2400
$\Sigma f = 50$					$\Sigma fd = -200$	$\Sigma fd^2 = 9400$

(b) standard deviation $\sigma = \sqrt{\frac{\Sigma fd^2}{\Sigma f} - \left(\frac{\Sigma fd}{\Sigma f}\right)^2}$

$$\sigma = \sqrt{\frac{9400}{50} - \left(\frac{-200}{50}\right)^2}$$

$$\sigma = \sqrt{188 - 16}$$

$$\sigma = \sqrt{172} = 13.11$$

(c) (i) Range = $60 - 11$
 $= 49 \text{ cm}$

(ii) Variance = $\sigma^2 = (\sqrt{172})^2 = 172$

Transcribe solution to paper

Question 10

(a) Area = $\int_1^2 (x^2 + 1) dx = \left[\frac{x^3}{3} + x \right]_1^2$

$$= \left[\frac{2^3}{3} + 2 \right] - \left[\frac{1^3}{3} + 1 \right]$$

$$= \left[\frac{8}{3} + 2 \right] - \left[\frac{1}{3} + 1 \right]$$

$$= \frac{8}{3} - \frac{1}{3} + 2 - 1$$

$$= \frac{7}{3} + 1 = 3\frac{1}{3} \text{ square units}$$

(b) (i) Given: $A = \begin{bmatrix} 4 & 2 \\ 5 & 1 \end{bmatrix}$

Det(A) = $4(1) - 5(2) = 4 - 10 = -6$

Cofactors of A = $\begin{bmatrix} 1 & -5 \\ -2 & 4 \end{bmatrix}$

Adjunct of A = $\begin{bmatrix} 1 & -2 \\ -5 & 4 \end{bmatrix}$

Inverse of A = $\frac{\text{Adjunct of A}}{\text{Det A}} = \frac{\begin{bmatrix} 1 & -2 \\ -5 & 4 \end{bmatrix}}{-6}$

$A^{-1} = \begin{bmatrix} -\frac{1}{6} & \frac{1}{3} \\ \frac{5}{6} & -\frac{2}{3} \end{bmatrix}$

(ii) $A^{-1} \times A = \begin{bmatrix} -\frac{1}{6} & \frac{1}{3} \\ \frac{5}{6} & -\frac{2}{3} \end{bmatrix} \begin{bmatrix} 4 & 2 \\ 5 & 1 \end{bmatrix}$

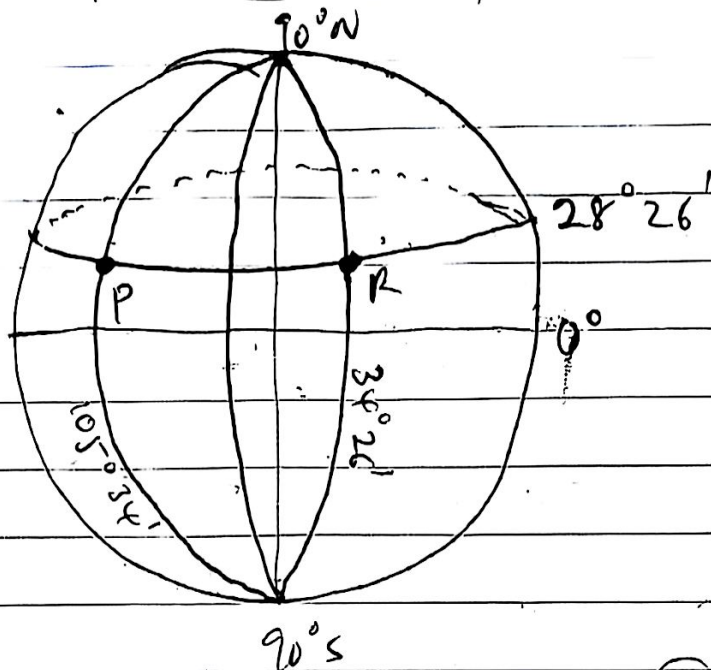
$$= \begin{bmatrix} (-\frac{1}{6} \times 4) + (\frac{1}{3} \times 5) & (-\frac{1}{6} \times 2) + (\frac{1}{3} \times 1) \\ (\frac{5}{6} \times 4) + (-\frac{2}{3} \times 5) & (\frac{5}{6} \times 2) + (-\frac{2}{3} \times 1) \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{2}{3} + \frac{5}{3} & -\frac{1}{3} + \frac{1}{3} \\ \frac{10}{3} - \frac{10}{3} & \frac{5}{3} - \frac{2}{3} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{3}{3} & 0 \\ 0 & \frac{3}{3} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

QUESTION 9



(a) $\bar{PR} = \frac{\theta}{360^\circ} \times 2\pi R \cos \phi$

$\bar{PR} = \frac{140^\circ}{360^\circ} \times 2 \times 314 \times 6400 \times \cos 28.43$

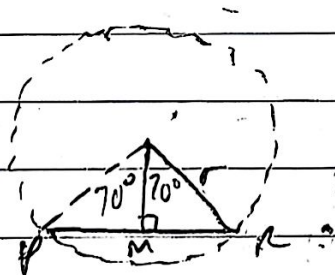
$= 13744.775$

$\approx 13700 \text{ km (3 s.f.)}$

where: $\theta = 105^\circ 34' + 34^\circ 26' = 140^\circ 00'$

$\phi = 28^\circ 26' = 28.43^\circ$

(b)



$\bar{PR} = r \sin 70^\circ = R \cos \phi \sin 70^\circ$

Length of chord, $\bar{PR} = 2R \cos \phi \sin 70^\circ$

$= 2 \times 6400 \times \cos 28.43 \times \sin 70^\circ$

$= 10577.14$

$= 10600 \text{ km (3 s.f.)}$