PAPER II

PART I

Answer all questions in this part.

1. (a) Find the distance between the points A(–3, 2) and B(4, – 6).

   (b) Find the acute angle between the lines $2x + 3y = 5$ and $3x + 2y = 1$ correct to one decimal place. **(8 marks)**

2. **Table 2.1:** Weights of Students

Weight (kg)	50	68	X	55	62
No. of students	3	5	1	2	4

   (a) If the mean weight of students in **Table 2.1** is 59.3 kg, determine the value of X to the nearest kg.

   (b) Calculate the:
   (i) Mode
   (ii) Median **(8 marks)**

3. (a) A box contains 10 mangoes and 9 bananas. If 2 fruits are picked from the box without replacement, find the probability that both are of the same fruits.

   (b) Use elimination method to solve the simultaneous equations;
   $2x + y = 17$, $3x + 2y = 2$.

   (8 marks)

4. Given that $9^{3x+1} = \frac{243^{x-3}}{3^x \times 81^{1-x}}$, find the value of x. **(8 marks)**

5. (a) Solve the equation $\frac{3}{a} = \frac{2}{a+1} + \frac{3}{3a-1}$

   (b) Simplify $\frac{x-y}{y+x} + \frac{y-x}{x+y}$

   (8 marks)

PART II

6. (a) Find the equation whose roots are $\frac{5}{6}$ and $\frac{-3}{4}$.

   (b) If α, α^2 are the roots of the equation $64x^2 + px + 27 = 0$, find the value of p.

   (c) Using completing the square method, solve the
   $25x^2 - 25x + 6 = 0$

7. (a) A function $y = 2x^4 - Px^3 + 2x$, where P is a constant, has a stationary value $x = -1$. Find the value of P.

(b) The motion of a particle along a straight line is defined by the equation $x = 5t^3 - 4t^2$. Find after 4 seconds, the:

(i) Velocity
(ii) Acceleration

(c) Evaluate $\int_0^3 6x^2 - 4x + 2\,dx$. **(12 marks)**

8. Two cities X and Y are on latitude 38°S and their longitudes are 50°E and 130°W respectively. Calculate and correct to 3 significant figures the:

(a) Speed of X due to the rotation of the earth
(b) Distance between X and Y along the great circle
(c) Distance between X and Y along the parallel of latitude
(Take $\pi = \frac{22}{7}$ and R = 6400 km)

(12 marks)

9. (a) Using a ruler and a pair of compasses only, construct:
(i) A triangle ABC in which |AB| = 8cm, ∠CAB = 60° and ∠ABC = 75°
(ii) The locus l_1 of points equidistant from A and B
(iii) The locus l_2 of points 6cm from A

(b) (i) Measure $|H_1H_2|$, where H_1 and H_2 are the points of intersection of l_1 and l_2.
(ii) Measure |AC|. **(12 marks)**

10. **Table 2.2** shows the scores of candidates in an entrance examination into a college.

Table 2.2: Scores of candidates

Marks(%)	21- 30	31 - 40	41 - 50	51 - 60	61 - 70	71 - 80	81 - 90
No. of students	9	20	31	42	50	32	16

(a) Prepare a cumulative frequency table and draw the cumulative frequency curve for the distribution.
(b) Use your curve to estimate the:
(i) Median mark
(ii) Upper and lower quartiles
(iii) Semi-inter quartile range

(12 marks)

11. (a) Copy and complete **Table 2.2** for the relation $y = \frac{1}{4}(2x^2 - 3x - 2)$.

Table 2.2: $y = \frac{1}{4}(2x^2 - 3x - 2)$.

x	−2	−1.5	−1	−0.5	0	1	1.5	2	2.5	3
y	3				−0.5			0	0.75	

(b) Draw the graph of $y = \frac{1}{4}(2x^2 - 3x - 2)$, using 2 cm to represent1 unit on both axes.

(c) From your graph, find the:

(i) Equation of the line of symmetry of the curve
(ii) Gradient of the curve at $x = 2.5$
(iii) Roots of the equation $\frac{1}{4}(2x^2 - 3x - 2) = 1$ **(12 marks)**

12. The length of 50 iron rods to the nearest cm are as follows:

13	15	22	27	41	45	54	20	23	11
31	23	35	13	45	18	39	23	12	27
54	26	60	36	53	21	42	31	25	35
16	37	27	23	55	33	41	26	58	39
18	28	35	12	49	25	43	16	11	22

(a) Prepare a frequency distribution table, using class intervals of 11-20, 21-30, 31- 40, ...

(b) Using an assumed mean of 35.5, compute the standard deviation.
(c) Find the:

i. Range of the distribution
ii. Variance **(12 marks)**

QUESTION 1

(a) Given: A(−3, 2) and B(4, −6)

$$\overline{AB} = \sqrt{(-6-2)^2 + (4-(-3))^2}$$

$$= \sqrt{(-8)^2 + (7)^2}$$

$$= \sqrt{64+49}$$

$$= \sqrt{113}$$

(b) Given: $2x+3y=5 \Rightarrow 3y = -2x+5$

$$y = -\tfrac{2}{3}x + \tfrac{5}{3}$$

gradient, $M_1 = -\tfrac{2}{3}$

Also: $3x+2y=1$

$$2y = -3x+1$$

$$y = -\tfrac{3}{2}x + \tfrac{1}{2}$$

gradient, $M_2 = -\tfrac{3}{2}$

$$\tan\theta = \frac{M_1 - M_2}{1+M_1M_2} = \frac{-\tfrac{2}{3} - (-\tfrac{3}{2})}{1+(-\tfrac{3}{2})(-\tfrac{2}{3})}$$

$$\tan\theta = \frac{5/6}{2} = \tfrac{5}{12}$$

$$\theta = \tan^{-1}(\tfrac{5}{12}) = 22.6^\circ$$

Acute angle = 22.6°

QUESTION 2

(a)

Weight (kg)	No of students	fx
50	3	150
68	5	340
x	1	x
55	2	110
62	4	248

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f} = \frac{150+340+x+110+248}{3+5+1+2+4}$$

$$= \frac{x+848}{15} = 59.3$$

$$x + 848 = 889.5$$

$$x = 889.5 - 848$$

$$x = 41.5$$

$$x = 42 \text{ kg}$$

(b) i Mode = 68 kg (highest frequency).

(ii) median = $\left(\frac{15+1}{2}\right)$th weight

$= \left(\frac{16}{2}\right)$th weight

$= 8$th weight

$= 68$ kg

QUESTION 3

(a) Total no of fruits = $10 + 9 = 19$

Prob (mango) = $\frac{10}{19}$ & prob (banana) = $\frac{9}{19}$

Prob (both same fruit) = Prob (both mango) + prob (both banana)

$$= \left(\frac{10}{19} \times \frac{9}{18}\right) + \left(\frac{9}{19} \times \frac{8}{18}\right)$$

$$= \frac{90}{342} + \frac{72}{342}$$

$$= 162/342 = 81/171$$

(b) $2x + y = 17$ ——— (1)

$3x + 2y = 2$ ——— (2)

multiply equ (1) by "2"

$4x + 2y = 34$ ——— (3)

substract equ (2) from equ (3)

$x = 32$

put $x = 32$ into equ (1)

$2(32) + y = 17$

$64 + y = 17$

$y = 17 - 64 = -47$

Question 4

$$9^{3x+1} = \frac{243^{x-2}}{3^x \times 81^{1-x}}$$

$$3^{2(3x+1)} = \frac{3^{5(x-2)}}{3^x \times 3^{4(1-x)}}$$

$$3^{6x+2} = \frac{3^{5x-10}}{3^{x+4-4x}}$$

$$3^{6x+2} = 3^{5x-10-x-4+4x}$$

$$3^{6x+2} = 3^{8x-14}$$

$$6x+2 = 8x-14$$

$$2+14 = 8x-6x$$

$$16 = 2x$$

$$x = 16/2 = 8$$

QUESTION 5

(a) $\frac{3}{a} = \frac{2}{a+1} + \frac{3}{3a-1}$

multiply through by L.C.M : $a(a+1)(3a-1)$

$3(a+1)(3a-1) = 2a(3a-1) + 3a(a+1)$

$3[3a^2+2a-1] = 6a^2-2a+3a^2+3a$

$9a^2+6a-3 = 9a^2+a$

$6a-3 = a$

$5a = 3$

$a = 3/5$

(b) $\frac{x-y}{y+x} \div \frac{y-x}{x+y} = \frac{x-y}{y+x} \times \frac{x+y}{y-x}$

$= \frac{x-y}{y-x}$

$= -\frac{(y-x)}{y-x}$

$= -1$

QUESTION 6

(a) Equation: $x^2 - (\text{sum of roots})x + (\text{product of roots}) = 0$

$x^2 - (5/6 - 3/4)x + (-3/4 \times 5/6) = 0$

$x^2 - \left(\frac{10-9}{12}\right)x + \left(-\frac{5}{8}\right) = 0$

$x^2 - (1/12)x - 5/8 = 0$

multiply through by 24

$24x^2 - 2x - 15 = 0$

(b) Given: $64x^2 + px + 27 = 0$

divide through by 64

$x^2 + p/64\,x + 27/64 = 0$

Sum of roots $= -p/64 = \alpha + \alpha^2$ ——— (1)

product of roots $= \frac{27}{64} = \alpha(\alpha^2)$ ——— (2)

From eq(2) $\alpha^3 = 27/64$

$\alpha = \sqrt[3]{27/64} = 3/4$

From equ (1) $-p/64 = 3/4 + (3/4)^2$

$-p/64 = \frac{3}{4} + \frac{9}{16}$

multiply through by 64

$-p = 16(3) + 4(9)$

$-p = 84$

$p = -84$

(c) $25x^2 - 25x + 6 = 0$

$x^2 - x + 6/25 = 0$

$x^2 - x = -6/25$

$(x - \frac{1}{2})^2 - \frac{1}{4} = -6/25$

$(x - 1/2)^2 = 1/4 - 6/25 = \frac{25-24}{100} = 1/100$

$(x - 1/2)^2 = 1/100$

$x - 1/2 = \pm 1/10$

$x = \frac{1}{2} \pm 1/10$

$x = \frac{1}{2} + 1/10$ or $x = \frac{1}{2} - 1/10$

$x = \frac{3}{5}$ or $x = \frac{2}{5}$

QUESTION 7

(a) Given: $y = 2x^4 - px^3 + 2x$

$$\frac{dy}{dx} = 8x^3 - 3px^2 + 2$$

Since stationary value $x = -1$

$$y'(-1) = 8(-1)^3 - 3p(-1)^2 + 2 = 0$$

$$-8 - 3p + 2 = 0$$

$$-3p = 8 - 2 = 6$$

$$p = 6/-3 = -2$$

(b) Given: $x = 5t^3 - 4t^2$

(i) velocity, $v = \frac{dx}{dt} = 15t^2 - 8t$

After 4 secs, $v = 15(4)^2 - 8(4)$

$= 240 - 32$

$= 208$ m/s

(ii) acceleration $= dv/dt = d/dt(15t^2 - 8t)$

$a = 30t - 8$

After 4 secs, $a = 30(4) - 8$

$a = 120 - 8$

$a = 112 \text{ m/s}^2$

(c) $\int_0^3 6x^2 - 4x + 2 \, dx = \left[2x^3 - 2x^2 + 2x\right]_0^3$

$= \left[2(3)^3 - 2(3)^2 + 2(3)\right] - 0$

$= 2(27) - 2(9) + 6$

$= 54 - 18 + 6$

$= 42$

QUESTION 8

(a) linear speed, v = angular speed × radius of latitude

$$v = \frac{2\pi}{24}\left(\frac{\text{radians}}{\text{hours}}\right) \times R\cos\phi$$

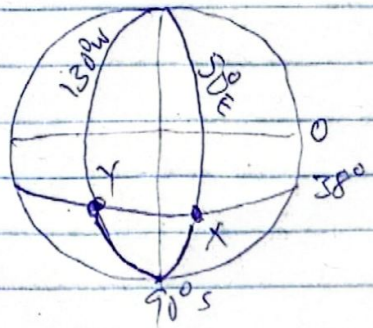
$$v = \frac{2\pi}{24} \times 6400 \times \cos 38^\circ \qquad \text{where } \phi = 38^\circ$$

$$v = \frac{2 \times 22}{24 \times 7} \times 6400 \times 0.7880$$

$$v = 1320.8 \text{ km/hr}$$

$$v = 1320 \text{ km/hr} \quad (3 \text{ s.f})$$

(b)



$\theta = 50 + 130 = 180^\circ$

Central angle between X and Y

$$\cos c = \sin\phi_1 \sin\phi_2 + \cos\phi_1 \cos\phi_2 \cos\theta$$

$$= \sin(-38)\sin(-38) + \cos(-38)\cos(-38)\cos 180^\circ$$

$$= (-0.6157)(-0.6157) + (0.788)(0.788)(-1)$$

$$= -0.2418$$

$$c = \cos^{-1}(-0.2418) = 104^\circ$$

$$\text{Distance} = \frac{c}{360} \times 2\pi R = \frac{104}{360} \times 2 \times \frac{22}{7} \times 6400$$

$$= 11620 \text{ km}$$

$$= 11600 \text{ km} \quad (3 \text{ s.f})$$

(c). Parallel of latitude

$$\overline{XY} = \frac{180}{360} \times \frac{2}{1} \times \frac{22}{7} \times 6400 \cos 38$$

$$= \frac{22}{7} \times 6400 \times 0.788 = 15850 \text{ km}$$

~~$= 11095 \text{ km}$~~

~~$= 11000 \text{ km} \quad (3 \text{ s.f})$~~

QUESTION 10

(a) Marks (%)	freq. f	Upper class boundary	Cumulative freq. c.f
21–30	9	30.5	9
31–40	20	40.5	29
41–50	31	50.5	60
51–60	42	60.5	102
61–70	50	70.5	152
71–80	32	80.5	184
81–90	16	90.5	200

(b) (i) median $= \frac{1}{2}(n+1)$th mark

$= \frac{1}{2}(200+1)$th mark

$= 100.5$th mark

median, $Q_2 = 60.5$ marks

(ii) lower quartile, $Q_1 = \frac{1}{4}(200+1)$th mark

$= 50.25$ mark

$Q_1 = 48$ marks

Upper quartile, $Q_3 = \frac{3}{4}(200+1)$ mark

$= 150.75$ mark

$Q_3 = 70.5$ marks

(iii) Semi-inter quartile range, $Q = \frac{Q_3 - Q_1}{2}$

$Q = \frac{70.5 - 48}{2}$

$Q = 11.25$

QUESTION 12

(a) ~~class interval~~ $A = 35.5$

Class Interval	frequency f	Midpoint x	$d = x - A$	d^2	fd	fd^2
11–20	12	15.5	−20	400	−240	4800
21–30	15	25.5	−10	100	−150	1500
31–40	10	35.5	0	0	0	0
41–50	7	45.5	10	100	70	700
51–60	6	55.5	20	400	120	2400
	$\Sigma f = 50$				$\Sigma fd = -200$	$\Sigma fd^2 = 9400$

(b) standard deviation $\sigma = \sqrt{\frac{\Sigma fd^2}{\Sigma f} - \left(\frac{\Sigma fd}{\Sigma f}\right)^2}$

$$\sigma = \sqrt{\frac{9400}{50} - \left(\frac{-200}{50}\right)^2}$$

$$\sigma = \sqrt{188 - 16}$$

$$\sigma = \sqrt{172} = 13.11$$

(c) (i) Range $= 60 - 11$
$= 49$ cm

(ii) Variance $= \sigma^2 = \left(\sqrt{172}\right)^2 = 172$