

PAPER II

PART I

Attempt all questions in this part.

1. Given that $9^{1x+1} = \frac{243^{x-2}}{3^x \times 81^{1-x}}$, find the value of x . (8 marks)
2. (a) Find the value of x if $\frac{9(x+4)-1}{10} = 8$.
 (b) If $x \propto y$ and $x = 8$ when $y = 4$, find the relationship between x and y . Hence, find y when $x = 10$. (8 marks)
3. (a) The square of the circumference of a circular track is $160,000 \text{ m}^2$. Find its radius to the nearest metre. (Take $\pi = \frac{22}{7}$) (8 marks)
 (b) The sides AB and AC of triangle ABC are produced to P and Q respectively such that $\angle PBC = 120^\circ$ and $\angle BCQ = 102^\circ$. Find $\angle BAC$. (8 marks)
4. Table 2.1 gives the weights (in kg) of 35 students in a particular school.

Table 2.1: Weights of students

Weight (kg)	40-44	45-49	50-54	55-59	60-64	65-69	70-74
Frequency	5	6	10	4	5	3	2

- (a) Construct a cumulative frequency table for the distribution.
- (b) Draw a cumulative frequency curve for the distribution.
- (c) Use your curve to estimate the median. (8 marks)
5. A particle moves in a straight line such that the distance travelled is specified by the equation $x = 2t^5 - 4t^4$. Find the:
 - (i) Velocity after 4 seconds
 - (ii) Acceleration after 4 seconds
 (8 marks)

PART II

Answer five questions only in this part.

6. (a) Given that $\log 2 = 0.3010$, $\log 3 = 0.4771$, $\log 5 = 0.6990$ and $\log 7 = 0.8451$, evaluate $\log 140 - \log \left(\frac{15}{2}\right)$ without using calculator or mathematical tables.
- (b) Solve for x if $\frac{3.6 \times 81^x}{1.2 \times 9^{x+1}} = \frac{1}{27}$. (12 marks)
7. (a) If $350x^2 = 225^2 - 125^2$, find the value of x without using calculator or mathematical tables.
- (b) The sum of the first six terms of an Arithmetic Progression is 34.5 and the sum of the next 6 terms is 123. Find the sum of the first term and the common difference. (12 marks)
8. (i) Chika is 5 years younger than Fatima and Kafaya is 7 years younger than the sum of the ages of Chika and Fatima. If the sum of their ages is 43. Find their ages.
- (ii) Given that $x + y = 7$ and $3x - y = 5$.
Show that $\frac{x}{3} + \frac{y}{4} = 2$. (12 marks)
9. P is a point on latitude $28^\circ 26' \text{ N}$ and longitude $105^\circ 34' \text{ W}$. R is on the same latitude as P but lies on longitude $34^\circ 26' \text{ E}$. Calculate, correct to three significant figures, the:
- (a) Distance between P and R along the latitude
- (b) Length of the chord PR
(Take $\pi = 3.14$, $R = 6,400 \text{ km}$) (12 marks)
10. (a) Find the area of the region bounded by the curve $y = x^2 + 1$, the ordinates $x = 1$, $x = 2$ and the x -axis.
- (b) (i) Find the inverse of the matrix $A = \begin{bmatrix} 4 & 2 \\ 5 & 1 \end{bmatrix}$.
- (ii) Evaluate $A^{-1} \times A$.
- (c) Given that $Q = \begin{pmatrix} 2 & 1 & 3 \\ 3 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix}$, find $|Q|$. (12 marks)

11. (a) Using a ruler and a pair of compasses only, construct:
- (i) A triangle ABC in which $|AB| = 8\text{cm}$,
 $\angle CAB = 60^\circ$ and $\angle ABC = 75^\circ$
 - (ii) The locus l_1 of points equidistant from A and B
 - (iii) The locus l_2 of points 6cm from A
- (b) (i) Measure $|H_1H_2|$, where H_1 and H_2 are the points of intersection of l_1 and l_2
- (ii) Measure $|AC|$. (12 marks)
12. (a) The probabilities that Uche, Precious and Godwin will pass with grade A_1 in Mathematics are $\frac{3}{8}$, $\frac{5}{7}$, $\frac{4}{5}$ respectively. Find the probability that:
- (i) Only one of them will have A_1
 - (ii) At least two of them will have A_1
 - (iii) None of them will have A_1
- (b) Olu tossed two fair dice together once. Draw a table to show the product of their outcomes. (12 marks)

QUESTION 2.

④

$$9(x+4) - 1 = 8$$

10

$$9(x+4) - 1 = 80$$

$$9(x+4) = 80 + 1 = 81$$

$$x+4 = 81/9 = 9$$

$$x+4 = 9$$

$$x = 9 - 4 = 5$$

⑤

$$x \propto y \Rightarrow x = ky$$

$$\text{if } x=8 \text{ \& } y=4$$

$$8 = 4k$$

$$k = 8/4 = 2$$

$$\text{Relationship } \Rightarrow x = 2y$$

$$\text{if } x = 10;$$

$$10 = 2y$$

$$y = 10/2 = 5$$

QUESTION 3

QUESTION 3

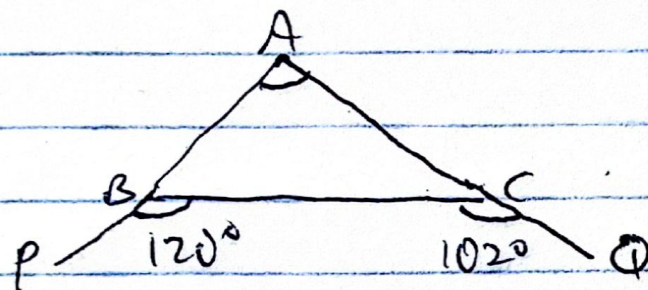
① Given: $(2\pi r)^2 = 160000$

$$2\pi r = \sqrt{160000} = 400$$

$$r = \frac{400}{2\pi} = \frac{400 \times 7}{2 \times 22} = \frac{700}{11}$$

$$\text{radius} = 63.6 = 64\text{m}$$

②



$$\hat{A}BC = 180^\circ - \hat{P}BC \text{ (sum of } \angle\text{s on a str. line} = 180^\circ)$$

$$\hat{A}BC = 180 - 120 = 60^\circ$$

$$\hat{A}CB = 180^\circ - \hat{B}CQ \text{ (sum of } \angle\text{s on a str. line} = 180^\circ)$$

$$\hat{A}CB = 180^\circ - 102 = 78^\circ$$

$$\hat{B}AC = 180^\circ - (\hat{A}BC + \hat{A}CB) \text{ (sum of } \angle\text{s in } \Delta = 180^\circ)$$

$$\hat{B}AC = 180^\circ - (60 + 78)$$

$$= 180 - 138$$

$$\hat{B}AC = 42^\circ$$

Question 4

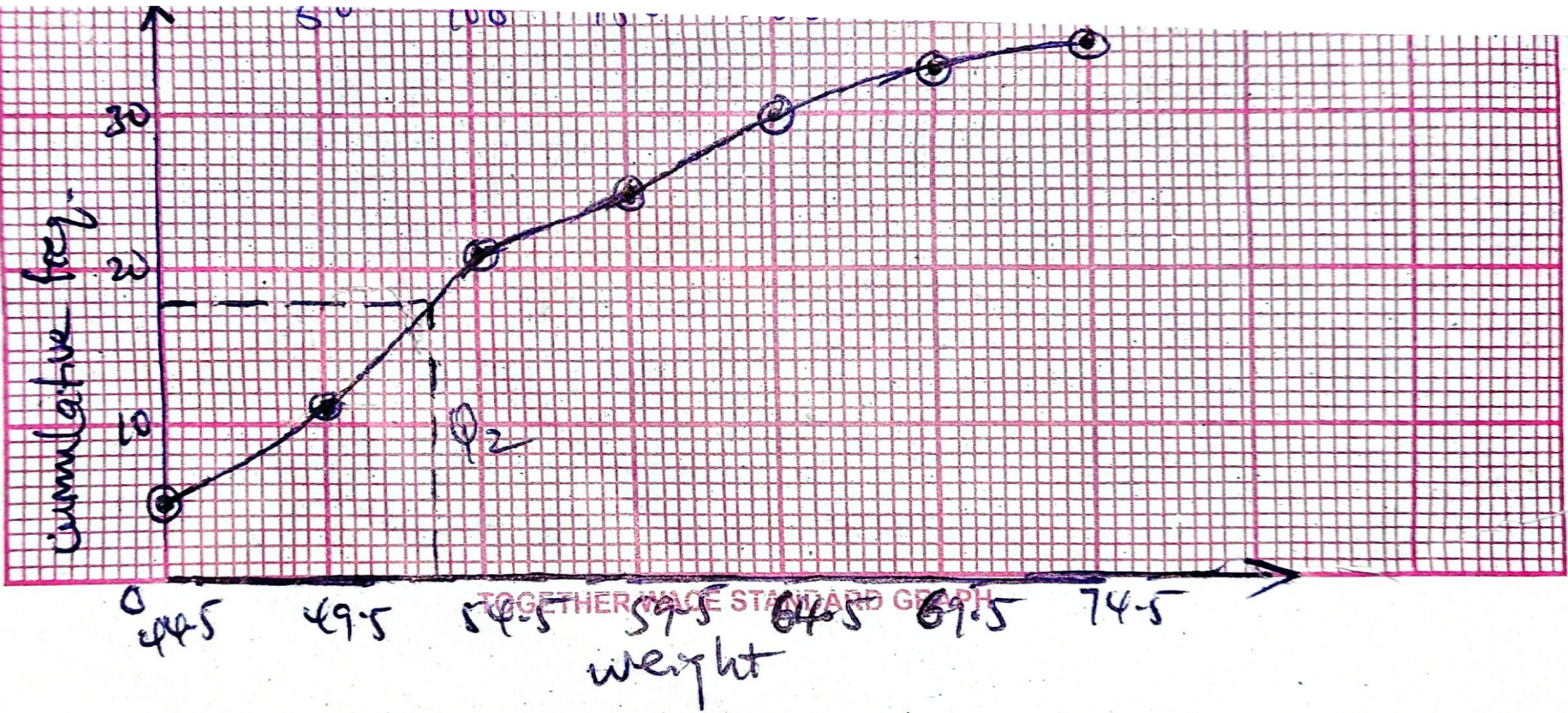
(a)

weight (kg)	freq. f	upper class boundary	Cumulative freq. - C.F
40-44	5	44.5	5
45-49	6	49.5	11
50-54	10	54.5	21
55-59	4	59.5	25
60-64	5	64.5	30
65-69	3	69.5	33
70-74	2	74.5	35

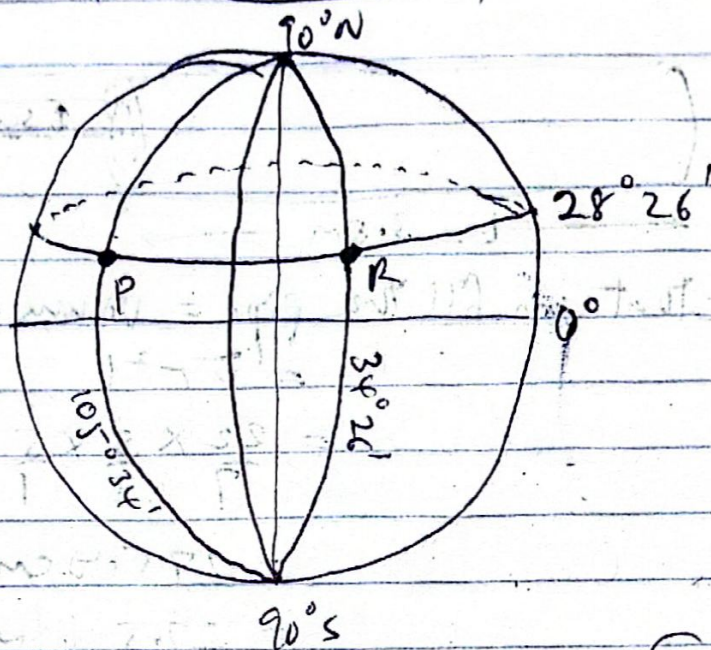
Frequency

(c) Median = $\frac{1}{2}(35+1)$ th position
 $\frac{1}{2}(36)$ th position
 18th mark

Median, $Q_2 = 53$ kg.



Question 9



(a) $\bar{PR} = \frac{\theta}{360^\circ} \times 2\pi R \cos \phi$

$\bar{PR} = \frac{140^\circ}{360^\circ} \times 2 \times 3.14 \times 6400 \times \cos 28.43^\circ$

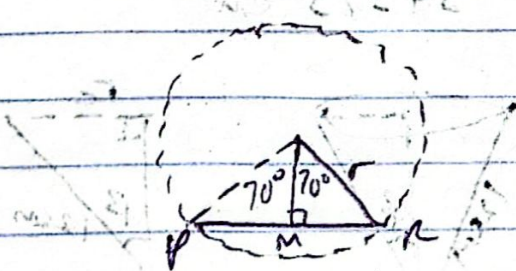
13744.775

$\approx 13700 \text{ km (3 s.f.)}$

where: $\theta = 105^\circ 34' + 34^\circ 26'$
 $140^\circ 00'$

$\phi = 28^\circ 26'$
 $= 28.43^\circ$

(b)



$\bar{MP} = r \sin 70^\circ = R \cos \phi \sin 70^\circ$

Length of chord, $\bar{PR} = 2R \cos \phi \sin 70^\circ$

$= 2 \times 6400 \times \cos 28.43^\circ \times \sin 70^\circ$

$= 10577.14$

$= 10600 \text{ km (3 s.f.)}$

Question 10

$$\begin{aligned}
 \textcircled{a} \text{ Area} &= \int_1^2 (x^2 + 1) dx = \left[\frac{x^3}{3} + x \right]_1^2 \\
 &= \left[\frac{2^3}{3} + 2 \right] - \left[\frac{1^3}{3} + 1 \right] \\
 &= \left[\frac{8}{3} + 2 \right] - \left[\frac{1}{3} + 1 \right] \\
 &= \frac{8}{3} - \frac{1}{3} + 2 - 1 \\
 &= \frac{7}{3} + 1 = 3\frac{1}{3} \text{ square units}
 \end{aligned}$$

$$\textcircled{b} \text{ (i) Given: } A = \begin{bmatrix} 4 & 2 \\ 5 & 1 \end{bmatrix}$$

$$\det(A) = 4(1) - 5(2) = 4 - 10 = -6$$

$$\text{Cofactors of } A = \begin{bmatrix} 1 & -5 \\ -2 & 4 \end{bmatrix}$$

$$\text{Adjunct of } A = \begin{bmatrix} 1 & -2 \\ -5 & 4 \end{bmatrix}$$

$$\text{Inverse of } A = \frac{\text{Adjunct of } A}{\det A} = \frac{\begin{bmatrix} 1 & -2 \\ -5 & 4 \end{bmatrix}}{-6}$$

$$A^{-1} = \begin{bmatrix} -\frac{1}{6} & \frac{1}{3} \\ \frac{5}{6} & -\frac{2}{3} \end{bmatrix}$$

$$\begin{aligned}
 \textcircled{ii} \text{ } A^{-1} \times A &= \begin{bmatrix} -\frac{1}{6} & \frac{1}{3} \\ \frac{5}{6} & -\frac{2}{3} \end{bmatrix} \begin{bmatrix} 4 & 2 \\ 5 & 1 \end{bmatrix} \\
 &= \begin{bmatrix} (-\frac{1}{6} \times 4) + (\frac{1}{3} \times 5) & (-\frac{1}{6} \times 2) + (\frac{1}{3} \times 1) \\ (\frac{5}{6} \times 4) + (-\frac{2}{3} \times 5) & (\frac{5}{6} \times 2) + (-\frac{2}{3} \times 1) \end{bmatrix} \\
 &= \begin{bmatrix} -\frac{2}{3} + \frac{5}{3} & -\frac{1}{3} + \frac{1}{3} \\ \frac{10}{3} - \frac{10}{3} & \frac{5}{3} - \frac{2}{3} \end{bmatrix} \\
 &= \begin{bmatrix} \frac{3}{3} & 0 \\ 0 & \frac{3}{3} \end{bmatrix} \\
 &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}
 \end{aligned}$$

QUESTION 8

(i) Let the ages of Chika, Fatma and Kafaya be c , f and k years respectively.

$$\Rightarrow f = c + 5 \quad \text{--- (1)}$$

$$\Rightarrow c + f = k + 7 \quad \text{--- (2)}$$

$$\Rightarrow c + f + k = 43 \quad \text{--- (3)}$$

Add equation (2) & equation (3)

$$2c + 2f = 50$$

$$f + c = 25 \quad \text{--- (4)}$$

Add equ (1) and equ (4)

$$2f = 30$$

$$f = \frac{30}{2} = 15$$

put this into equ (1)

$$15 = c + 5$$

$$c = 15 - 5 = 10$$

put the values of f and c into equ (3)

$$10 + 15 + k = 43$$

$$k = 43 - 25$$

$$k = 18$$

Chika is 10 years; Fatma is 15 years old and Kafaya is 18 years old.

$$(ii) \quad x + y = 7 \quad \text{--- (1)}$$

$$3x - y = 5 \quad \text{--- (2)}$$

Add up

$$4x = 12$$

$$x = \frac{12}{4} = 3$$

Put this in equ (1)

$$3 + y = 7$$

$$y = 7 - 3 = 4$$

$$\frac{3}{3} + \frac{4}{4} = \frac{3}{3} + \frac{4}{4}$$

$$= 1 + 1$$

$$= 2$$

shown

QUESTION 5

Given: $x = 2t^5 - 4t^4$

(i) velocity, $v = \frac{dx}{dt} = 2(5)t^4 - 4(4)t^3$
 $= 10t^4 - 16t^3$

After 4 secs,

$$v = 10(4)^4 - 16(4)^3$$

$$v = 10(256) - 16(64)$$

$$v = 2560 - 1024$$

$$v = 1536 \text{ m/s}$$

Thinker solution for

(ii) Acceleration, $a = \frac{dv}{dt} = \frac{d}{dt}(10t^4 - 16t^3)$

$$= 10(4)t^3 - 16(3)t^2$$

$$a = 40t^3 - 48t^2$$

After 4 secs, $a = 40(4)^3 - 48(4)^2$

$$= 2560 - 768$$

$$= 1792 \text{ m/s}^2$$

Question 6

$$\begin{aligned}
 \textcircled{a} \quad & \log 140 - \log \left(\frac{15}{2} \right) \\
 &= \log (7 \times 2 \times 10) - [\log 15 - \log 2] \\
 &= \log 7 + \log 2 + \log 10 - \log 15 + \log 2 \\
 &= \log 7 + 2\log 2 + \log 10 - \log 15 \\
 &= \log 7 + 2\log 2 + \log 10 - \log 3 - \log 5 \\
 &= 0.8451 + 2(0.3010) + 1.0000 - 0.4771 - 0.6990 \\
 &= 2.4471 - 1.1761 \\
 &= 1.271
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{b} \quad & \frac{3.6 \times 81^x}{1.2 \times 9^{2x+1}} = \frac{1}{27} \\
 & \left[\frac{3.6}{1.2} \times \frac{81^x}{9^x(9)} \right] = \frac{1}{27} \\
 & \frac{3(9^x)^2}{9^x(9)} = \frac{1}{27} \\
 & \frac{9^x}{3} = \frac{1}{27} \\
 & 9^x = \frac{3}{27} = \frac{1}{9} \\
 & 9^x = 9^{-1} \\
 & x = -1
 \end{aligned}$$

QUESTION

$$9) 350x^2 = 225^2 - 125^2$$

$$350x^2 = (225+125)(225-125)$$

$$350x^2 = (350)(100)$$

$$x^2 = \frac{(350)(100)}{350}$$

$$x^2 = 100$$

$$x = \sqrt{100} = \pm 10$$

diff of two squares

$$b) S_n = \frac{n}{2} [2a + (n-1)d] \text{ for A.P.}$$

$$S_6 = \frac{6}{2} [2a + 5d] = 34.5$$

$$2a + 5d = 11.5 \quad \text{--- (1)}$$

Also

$$S_{12} = \frac{12}{2} [2a + 11d] = 34.5 + 123$$

$$6 [2a + 11d] = 157.5$$

$$2a + 11d = 26.25 \quad \text{--- (2)}$$

Subtract equ (1) from equ (2)

$$6d = 26.25 - 11.5$$

$$6d = 14.75$$

$$d = 14.75/6 = \frac{59}{24}$$

Put this in to equ (1)

$$2a + 5\left(\frac{59}{24}\right) = \frac{23}{2}$$

$$2a = \frac{23}{2} - \frac{295}{24} = \frac{-19}{24}$$

$$a = \frac{-19}{48}$$

$$\text{Sum of first and common difference} = \frac{59}{24} - \frac{19}{48}$$

$$= \frac{118}{48} - \frac{19}{48}$$

$$= \frac{99}{48}$$

$$= \frac{33}{16}$$