PAPER II

PART I

Answer **all** questions in this part.

1. (a) Out of 930 students in a college, 450 are girls. $\frac{2}{3}$ of the girls and $\frac{3}{4}$ of the boys are less than 20 years old. If a student is picked at random, find the probability that the student is less than 20 years.

   (b) There are 6 red, 4 blue and 8 green balls in a bag. Two balls are picked at random one after the other without replacement. Find the probability that the two balls are either red or blue. **(8 marks)**

2. (a) A particle moves along x-axis so that its distance x metres from the origin after t seconds is given by $x = 7t + 12t^2$. Find its:

   (i) Velocity after 2 seconds
   (ii) Acceleration

   (b) Simplify $\frac{m^2 - 2mn + n^2}{m^2 - n^2}$. **(8 marks)**

3. (a) Calculate the simple interest on ₦720,000.00 for 3 years at $5\frac{1}{2}\%$ per annum. Hence, find the amount at the end of 3 years.

   (b) In a class of 75 students, the ratio of boys to girls is 2:3. How many boys should be added to the class in order to get a ratio 7:9?

   (8 marks)

4. (a) Find the value of x if $\frac{9(x+4)-1}{10} = 8$.

   (b) If $x \propto y$ and $x = 8$ when $y = 4$, find the relationship between x and y. Hence, find y when $x = 10$. **(8 marks)**

5. (a) The square of the circumference of a circular track is 160,000 m^2. Find its radius to the nearest metre. (Take $\pi = \frac{22}{7}$) **(8 marks)**

   (b) The sides AB and AC of triangle ABC are produced to P and Q respectively such that $\angle PBC = 120^\circ$ and $\angle BCQ = 102^\circ$. Find $\angle BAC$. **(8 marks)**

PART II

6. In **Fig. 2.1**, OAB is a sector of a circle with centre O and radius 16 cm.

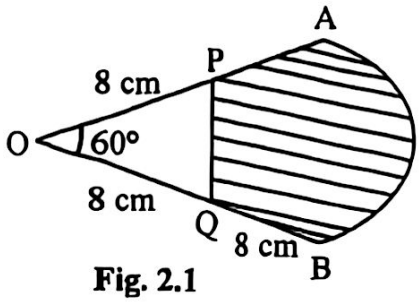


Fig. 2.1

(a) Find the area of the shaded portion.

(b) Calculate the perimeter of the shaded portion.
(Take $\pi = \frac{22}{7}$) **(12 marks)**

7. (a) Towns **P**(lat. 40°N, long.20°W) and **Q**(lat.40°N, long.70°W) are two places on the Earth's surface. Calculate, correct to 2 decimal places, the:
(i) Radius of the parallel of latitude on which **P** and **Q** lie
(ii) Shortest distance **PQ** along the parallel of latitude
(iii) Time taken for an aeroplane to cover the shortest distance **PQ** at an average speed of 800 km/h. (Use $\pi = \frac{22}{7}$ and R = 6,400 km)

(b) Simplify $\frac{2(x^2+x-6)}{3x^2-27} + \frac{x^2-x-2}{x-3}$. **(12 marks)**

8. (a) Copy and complete **Table 2.1** for the relation $y = 2x^2 - 5x - 6$, for $-2 \leq x \leq 5$.

Table 2.1: Values of x *and* y

x	−2	−1	0	1	2	3	4	5
y	12					−3		19

(b) Using a scale of 2 cm to represent 1 unit on x-axis and 2 cm to represent 5 units on y-axis, draw the graph of $y = 2x^2 - 5x - 6$ for $-2 \leq x \leq 5$.

(c) Use your graph to find the:

(i) Roots of the equation $2x^2 - 5x - 6 = 0$, correct to one decimal place
(ii) Values of x for which $2x^2 - 5 = 8$, correct to one decimal place
(iii) Gradient of the tangent at the point where $x = 2$

(12 marks)

9. (a) The probabilities that Uche, Precious and Godwin will pass with grade A_1 in Mathematics are $\frac{3}{8}$, $\frac{5}{7}$, $\frac{4}{5}$ respectively.
Find the probability that:
(i) Only one of them will have A_1
(ii) At least two of them will have A_1
(iii) None of them will have A_1

(b) Olu tossed two fair dice together once.
Draw a table to show the product of their outcomes.

(12 marks)

10. (a) If $T = 2\pi\sqrt{\frac{L}{G}}$, make G the subject of the formula.
Hence, find the value of G to the nearest whole number when $T = 45$, $L = 8$ and $\pi = 3.142$.

(b) In a particular guest house, the cost per person (C) for an accommodation is partly constant and partly varies inversely as the number of guests (N). If the cost per person, for 24 persons is ₦800.00 and the cost per person, for 18 persons is ₦1200.00, find the cost per person, for 42 persons to the
nearest naira.

(12 marks)

11. (a) A cylindrical metal pipe 3.5 m long has internal and external radii 5.0 cm and 7.5 cm respectively. Calculate the volume of:
(i) Water that can fill the pipe in litres
(ii) Metal in the pipe in cm^3

(b) A sector of a circle with radius 12 cm and angle 240° is used to form a cone. Calculate the:
(i) Radius of the cone
(ii) Vertical angle of the cone, correct to two decimal places

(12 marks)

12. P is a point on latitude 28° 26' N and longitude 105° 34' W. R is on the same latitude as P but lies on longitude 34° 26' E. Calculate, correct to three significant figures, the:
(a) Distance between P and R along the latitude
(b) Length of the chord PR
(Take $\pi = 3.14$, R = 6,400 km)

(12 marks)

QUESTION 1

(a) Total no of students = 930

No of girls = 450 and no of boys = 930 − 450 = 480

$\frac{2}{3}$ of the girls = $\frac{2}{3} \times 450 = 300$

$\frac{3}{4}$ of boys = $\frac{3}{4} \times 480 = 360$

Tot of students less than 20 years = 300 + 360 = 660

Prob (< 20 years) = $\frac{660}{930} = \frac{22}{31}$

(b) Total no of balls = 6 + 4 + 8 = 18 balls

Prob (red) = $\frac{6}{18}$; prob (blue) = $\frac{4}{18}$; prob (green) = $\frac{8}{18}$

Prob (either red or blue) = [1st red & 2nd blue] + [1st blue & 2nd red]

$$= \left[\frac{6}{18} \times \frac{4}{17}\right] + \left[\frac{4}{18} \times \frac{6}{17}\right]$$

$$= \left[\frac{1}{3} \times \frac{4}{17}\right] + \left[\frac{1}{3} \times \frac{4}{17}\right]$$

$$= \frac{4}{51} + \frac{4}{51}$$

$$= \frac{8}{51}$$

QUESTION 2

(a) Given: $x = 7t + 12t^2$

(i) Velocity, $v = \frac{dx}{dt} = 7 + 12(2)t$

$= 7 + 24t$

After 2 secs, $v = 7 + 24(2)$

$= 7 + (48)$

$= 55 \text{ m/s}$

(ii) Acceleration, $a = dv/dt = \frac{d}{dt}(7 + 24t)$

$= 24 \text{ m/s}^2$

(b) $\frac{m^2 - 2mn + n^2}{(m^2 - n^2)} = \frac{m^2 - mn - mn + n^2}{(m+n)(m-n)}$

$= \frac{m(m-n) - n(m-n)}{(m+n)(m-n)}$

$= \frac{(m-n)(m-n)}{(m+n)(m-n)}$

$= \frac{m-n}{m+n}$

QUESTION 3

(a) Simple interest $= \frac{PRT}{100} = \frac{₦720,000 \times 5.5 \times 3}{100}$

$I = ₦118,800$

Amount = Principal + interest

$= ₦720,000 + ₦118,000$

$= ₦838,800.00$

(b) No of boys $= \frac{2}{5} \times 75 = 2 \times 15 = 30$ boys

Hence: there are $75 - 30 = 45$ girls

$\frac{P+30}{45} = \frac{7}{9}$

$P + 30 = \frac{7}{9} \times 45 = 35$

$P = 35 - 30 = 5$

5 boys must be added.

QUESTION 4

(a) $\frac{9(x+y)-1}{10} = 8$

$9(x+y)-1 = 80$

$9(x+y) = 80+1 = 81$

$x+y = 81/9 = 9$

$x+y = 9$

$x = 9-4 = 5$

(b) $x \propto y \Rightarrow x = ky$

if $x=8$ & $y=4$

$8 = 4k$

$k = 8/4 = 2$

Relationship $\Rightarrow x = 2y$

if $x = 10$;

$10 = 2y$

$y = 10/2 = 5$

QUESTION 5

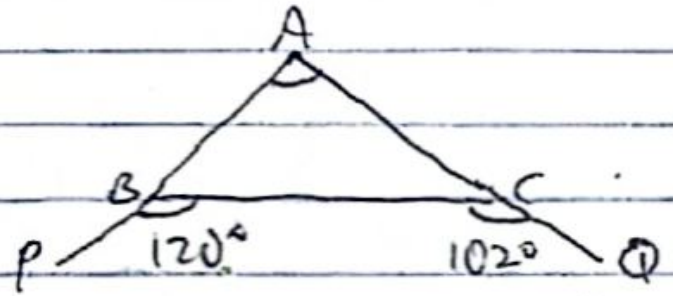
(a) Given: $(2\pi r)^2 = 160\,000$

$2\pi r = \sqrt{160\,000} = 400$

$r = \frac{400}{2\pi} = \frac{400 \times 7}{2 \times 22} = \frac{700}{11}$

radius $= 63.6 = 64\text{m}$

(b)



$A\hat{B}C = 180° - P\hat{B}C$ (sum of $\angle$s on a str. line $= 180°$)

$A\hat{B}C = 180 - 120° = 60°$

$A\hat{C}B = 180° - B\hat{C}Q$ (sum of $\angle$s on a str. line $= 180°$)

$A\hat{C}B = 180° - 102 = 78°$

$B\hat{A}C = 180° - (A\hat{B}C + A\hat{C}B)$ (sum of $\angle$s in a $\Delta = 180°$)

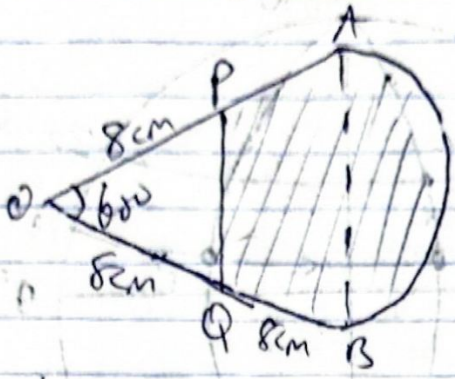
$B\hat{A}C = 180° - (60 + 78)$

$= 180 - 138$

$B\hat{A}C = 42°$

QUESTION 6

(a)



Area of sector OAB $= \frac{\theta}{360} \times \pi r^2$

$= \frac{60}{360} \times \frac{22}{7} \times \frac{16}{1} \times \frac{16}{1}$

$= \frac{2816}{21}\ cm^2$

Area of triangle OPQ $= \frac{1}{2} \sin\theta \times |OP| \times |OQ|$

$= \frac{1}{2} \times \sin 60^\circ \times 8^2$

$= 32 \times \frac{\sqrt{3}}{2} = 16\sqrt{3}$

Area of shaded portion $= \frac{2816}{21} - 16\sqrt{3}$

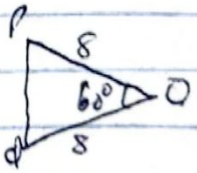
$= \frac{2816 - 336\sqrt{3}}{21}$

$= 106.38\ cm^2$

(b) Arc AB $= \frac{\theta}{360} \times 2\pi r$

$= \frac{60}{360} \times \frac{2}{1} \times \frac{22}{7} \times \frac{16}{1}$

$= \frac{352}{21}\ cm$



Perimeter of shaded portion $= 8 + 8 + \frac{352}{21} + \overline{PQ}$

$= \frac{688}{21} + \overline{PQ}$

Using cosine: $\overline{PQ} = \sqrt{8^2 + 8^2 - 2(8)(8)\cos 60^\circ}$

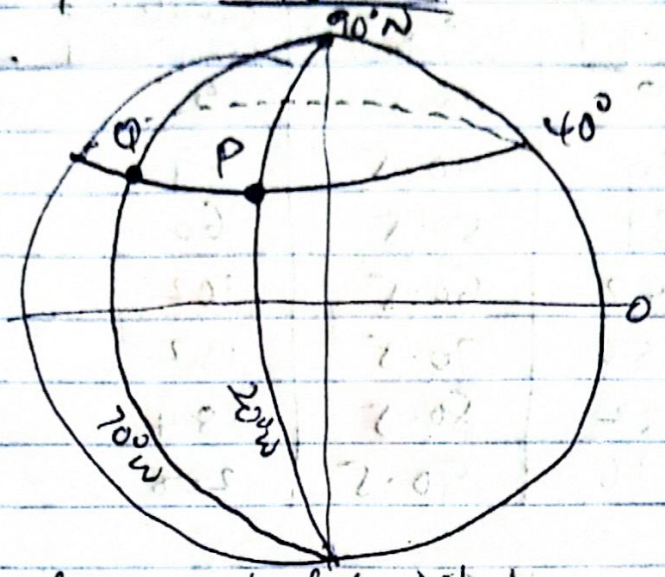
$= \sqrt{64}$

$\overline{PQ} = 8$

$\therefore$ Perimeter $= \frac{688}{21} + 8 = \frac{856}{21} = 40.76\ cm$

QUESTION 17

(a)



(i) Radius of parallel of latitude, $r = R\cos\phi$

$= 6400 \times \cos 40^\circ$

$= 4902.68$ km

(ii) $\overline{PQ} = \frac{\theta}{360} \times 2\pi r$

$= \frac{70-20}{360} \times \frac{2}{1} \times \frac{22}{7} \times 4902.68$

$= 4280.12$ km

(iii) time, $t = \frac{\text{distance } \overline{PQ}}{\text{speed}} = \frac{4280.12}{800} = 5.35$ hours

(b) $\frac{2(x^2+x-6)}{3x^2-27} \div \frac{x^2-x-2}{x-3}$

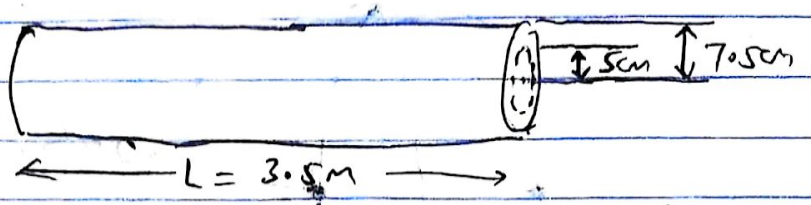
$= \frac{2(x+3)(x-2)}{3(x^2-9)} \div \frac{(x-2)(x+1)}{x-3}$

$= \frac{2(x+3)(x-2)}{3(x+3)(x-3)} \times \frac{(x-3)}{(x-2)(x+1)}$

$= \frac{2}{3(x+1)}$

QUESTION 11

(a)



(ii) water that can fill the pipe = volume of inner cylinder

$= \pi r^2 L$

$= \frac{22}{7} \times \frac{5}{1} \times \frac{5}{1} \times 350$

$= 27500\ cm^3$

$= 27.5$ litres

(iii) Volume of metal in the pipe $= \pi (R^2 - r^2) L$

$= \frac{22}{7} \times (7.5^2 - 5.0^2) \times 350$

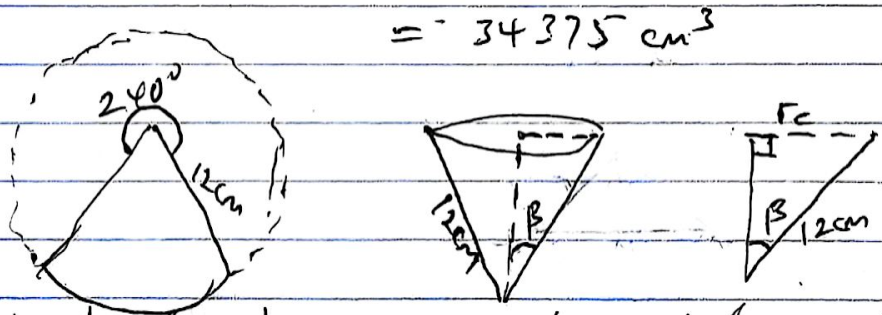
$= \frac{22}{7} \times (7.5+5.0)(7.5-5.0) \times 350$

$= \frac{22}{7} \times 12.5 \times 2.5 \times 350$

$= 34375\ cm^3$

(b)

(i)



Arc length of sector = circular part of cone

$\frac{\theta}{360} \times 2\pi r_s = 2\pi r_c$

$\frac{240}{360} \times 2 \times 12 = 2r_c$

$16 = 2r_c$

radius of cone, $r_c = 16/2 = 8cm$

NB: radius of sector = slant height of cone

(ii) Vertical angle $2\beta = 2 \times \sin^{-1}(r_c/l)$

$= 2 \times \sin^{-1}(8/12)$

$= 2 \times \sin^{-1}(2/3)$

$= 2 \times 41.81$

$= 83.62^\circ$

QUESTION 10

(a)

$$T = 2\pi\sqrt{\frac{L}{G}}$$

Square both sides

$$T^2 = 4\pi^2 \frac{L}{G}$$

$$G = \frac{4\pi^2 L}{T^2}$$

when: $T = 45$, $L = 8$ and $\pi = 3.142$

$$G = \frac{4 \times (3.142)^2 \times 8}{45^2}$$

$$G = 0.156$$

$G \approx 0$ (nearest whole number)

(b) Given: $C = a + \frac{b}{N}$ where a & b are constants

Given: $C = ₦800$ and $N = 24$

$$800 = a + \frac{b}{24} \quad \text{——— (1)}$$

Given: $C = ₦1200$ and $N = 18$

$$1200 = a + \frac{b}{18} \quad \text{——— (2)}$$

Subtract equ (1) from equ (2)

$$400 = b\left(\frac{1}{18} - \frac{1}{24}\right)$$

$$400 = b\left(\frac{4-3}{72}\right)$$

$$400 = \frac{b}{72}$$

$$b = 28800$$

put this in equ (1)

$$800 = a + \frac{28800}{24}$$

$$a = 800 - 1200$$

$$a = -400$$

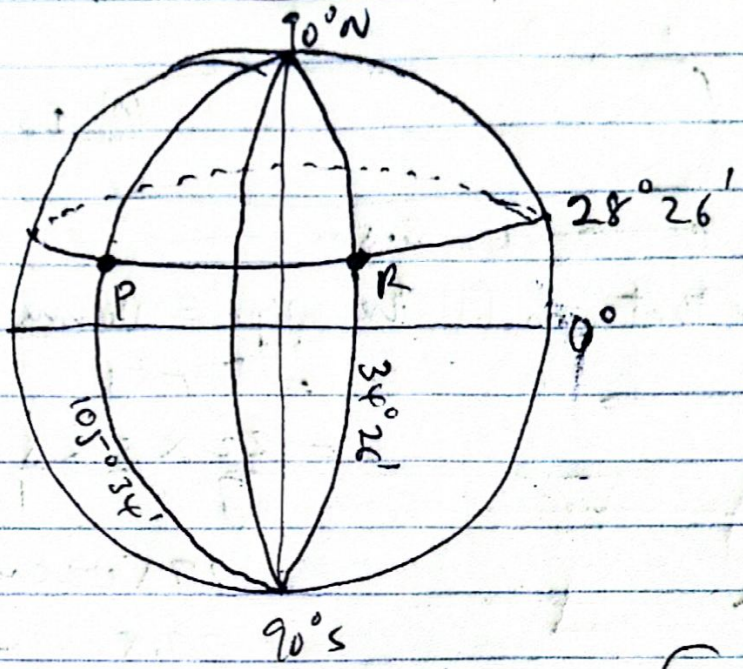
Equation: $C = -400 + \frac{28800}{N}$

When: $N = 42$

$$C = -400 + \frac{28800}{42} = -400 + 685.71$$

$$= ₦286 \text{ (nearest naira)}$$

QUESTION 12



(a) $\overline{PR} = \frac{\theta}{360^\circ} \times 2\pi R \cos\phi$

$\overline{PR} = \frac{140^\circ}{360^\circ} \times 2 \times 3.14 \times 6400 \times \cos 28.43$

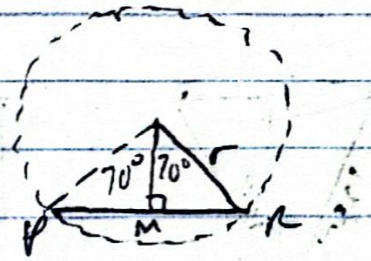
$= 13744.775$

$= 13700 km (3 s.f)$

where: $\theta = 105^\circ 34' + 34^\circ 26' = 140^\circ 00'$

$\phi = 28^\circ 26' = 28.43^\circ$

(b)



$\overline{MR} = r \sin 70^\circ = R\cos\phi \sin 70^\circ$

Length of chord, $\overline{PR} = 2R\cos\phi \sin 70^\circ$

$= 2 \times 6400 \times \cos 28.43 \times \sin 70^\circ$

$= 10577.14$

$= 10600 km (3.s.f)$